Within-cases analysis of Normal Data[*]

The grapefruit data

Grapefruit sales were measured for eight grocery stores at three price levels. The multivariate data format is more natural, but `lmer` wants each value of the response variable to be on a separate line.

Multivariate format

```
 Store    sales1    sales2    sales3
   1       62.1      61.3      60.8
   2       58.2      57.9      55.1
   3       51.6      49.2      46.2
   4       53.7      51.5      48.3
   5       61.4      58.7      56.6
   6       58.5      57.2      54.3
   7       46.8      43.2      41.5
   8       51.2      49.8      47.9
```

Univariate format

```
 Store    Price    Sales
   1       1       62.1
   1       2       61.3
   1       3       60.8
   2       1       58.2
   2       2       57.9
   2       3       55.1
   3       1       51.6
   3       2       49.2
   3       3       46.2
   4       1       53.7
   4       2       51.5
   4       3       48.3
   5       1       61.4
   5       2       58.7
   5       3       56.6
   6       1       58.5
   6       2       57.2
   6       3       54.3
   7       1       46.8
   7       2       43.2
   7       3       41.5
   8       1       51.2
   8       2       49.8
   8       3       47.9
```

These data are from a homework problem in *Applied linear statistical models* (2005, 5th edition)

* Copyright information is on the last page.

by Kutner et al.

```
> ###################### Grapefruit ######################
> # http://www.utstat.utoronto.ca/~brunner/data/illegal/GrapefruitData.html
>
> rm(list=ls()); options(scipen=999) # To avoid scientific notation
> # Install packages if necessary. Only need to do this once.
> # install.packages("lme4")
> # install.packages("car")
> # Load packages -- do this every time
> library(lme4) # For lmer function
Loading required package: Matrix
> library(car)  # For F-tests, likelihood ratio and Wald chi-squared tests
> # Avoid the lmerTest package. It seems to have actual errors.
>
> fruit = read.table("grapefruit.data.txt",header=T) # Local copy
> head(fruit); attach(fruit)
  Store Price Sales
1     1     1  62.1
2     1     2  61.3
3     1     3  60.8
4     2     1  58.2
5     2     2  57.9
6     2     3  55.1
> table(Price,Store)
     Store
Price 1 2 3 4 5 6 7 8
    1 1 1 1 1 1 1 1 1
    2 1 1 1 1 1 1 1 1
    3 1 1 1 1 1 1 1 1
> Price = factor(Price) # Otherwise it's numeric
> aggregate(fruit,by=list(Price),FUN=mean)
  Group.1 Store Price   Sales
1       1   4.5     1 55.4375
2       2   4.5     2 53.6000
3       3   4.5     3 51.3375
> gfmixed = lmer(Sales ~ Price + (1 | Store) )
> summary(gfmixed)
Linear mixed model fit by REML ['lmerMod']
Formula: Sales ~ Price + (1 | Store)

REML criterion at convergence: 93.2

Scaled residuals:
     Min       1Q   Median       3Q      Max
-1.48527 -0.41034 -0.07546  0.53703  1.90090

Random effects:
 Groups   Name        Variance Std.Dev.
 Store    (Intercept) 35.2571  5.9378
 Residual              0.6837  0.8269
Number of obs: 24, groups:  Store, 8

Fixed effects:
            Estimate Std. Error t value
(Intercept)  55.4375     2.1196  26.155
Price2       -1.8375     0.4134  -4.444
Price3       -4.1000     0.4134  -9.917

Correlation of Fixed Effects:
       (Intr) Price2
Price2 -0.098
```

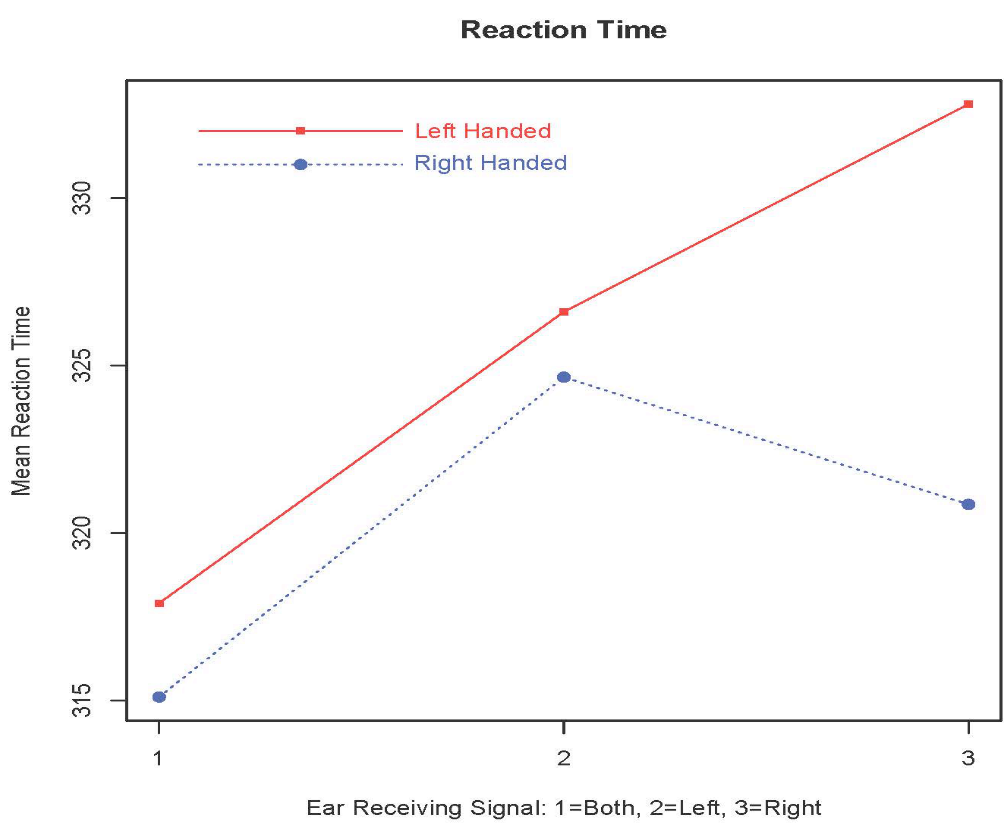
```
Price3 -0.098  0.500

> anova(gfmixed)
Analysis of Variance Table
      Df Sum Sq Mean Sq F value
Price  2 67.481   33.74  49.346
> # No p-values
> Anova(gfmixed, test="F") # F test (from car package)
Analysis of Deviance Table (Type II Wald F tests with Kenward-Roger df)

Response: Sales
           F Df Df.res       Pr(>F)
Price 49.346  2     14 0.0000004567 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
>
> # Compare naive fixed effects model
> anova(lm(Sales ~ Price))
Analysis of Variance Table

Response: Sales
          Df Sum Sq Mean Sq F value Pr(>F)
Price      2  67.48  33.740  0.9388 0.4069
Residuals 21 754.76  35.941
```

Dichotic listening

Left-handed and right-handed subjects push a key when they hear their names over background noise. They are wearing stereo headphones. The signal comes in the left ear, the right ear, or both. There are 50 trials in each condition, presented in a different random order for each subject. The response variable is median reaction time in milliseconds. Each subject contributes 3 medians.

```
> rm(list=ls()); options(scipen=999) # To avoid scientific notation
> # Install packages if necessary. Only need to do this once.
> # install.packages("lme4")
> # install.packages("car")
> # Load packages -- do this every time
> library(lme4) # For lmer function
Loading required package: Matrix
> library(car)  # For F-tests, likelihood ratio and Wald chi-squared tests
> # Avoid the lmerTest package. It seems to have actual errors.
>
> # Read data into a data frame
> dichotic =
read.table("http://www.utstat.toronto.edu/~brunner/data/legal/HandEar.data.txt")
> head(dichotic); attach(dichotic)
  subject handed   ear rtime
1       1   Left  Left   330
2       1   Left Right   327
3       1   Left  Both   303
4       2   Left  Left   294
5       2   Left Right   339
6       2   Left  Both   315
>
> # Sample sizes
> table(handed,ear)
       ear
handed  Both Left Right
  Left    20   20    20
  Right   20   20    20
>
> meantable = aggregate(rtime,by=list(ear,handed),FUN=mean)
> colnames(meantable) = c("Ear","Handed","Mean RT")
> meantable
    Ear Handed Mean RT
1  Both   Left  317.90
2  Left   Left  326.60
3 Right   Left  332.80
4  Both  Right  315.10
5  Left  Right  324.65
6 Right  Right  320.85
>
> # Two-way table of means
> LeftHanded = meantable[1:3,3]; RightHanded =  meantable[4:6,3]
> TwoWay = rbind(LeftHanded,RightHanded)
> colnames(TwoWay) = c("BothEars","LeftEar","RightEar")
> TwoWay
            BothEars LeftEar RightEar
LeftHanded     317.9  326.60   332.80
RightHanded    315.1  324.65   320.85
```

```
>
> addmargins(TwoWay,margin=c(1,2),FUN=mean) # Add marginal means to table
Margins computed over dimensions
in the following order:
1:
2:
            BothEars LeftEar RightEar     mean
LeftHanded     317.9 326.600  332.800 325.7667
RightHanded    315.1 324.650  320.850 320.2000
mean           316.5 325.625  326.825 322.9833
>
> # Plot the means
> Means = meantable[,3] # All the rows, column 3
> lhand = Means[1:3]; rhand = Means[4:6]
> Ear = c(1:3,1:3)
> # Invisible points at first, x axis points at 1,2,3; see help(plot)
> plot(Ear,Means,pch=" ",xaxp=c(1,3,2),
+     xlab="Ear Receiving Signal: 1=Both, 2=Left, 3=Right",
+     ylab="Mean Reaction Time")
> title("Reaction Time")
> points(1:3,lhand,col='red',pch=15) # Red squares
> points(1:3,rhand,col='blue',pch=19) # Blue circles
> lines(1:3,lhand,lty=1,col='red'); lines(1:3,rhand,lty=3,col='blue')
> # Annotate the plot
> x1 = c(1.1,1.6); y1 = c(332,332); lines(x1,y1,lty=1,col='red')
> points(1.35,332,col='red',pch=15)
> text(1.80,332,'Left Handed',col='red')
> x2 = c(1.1,1.6); y2 = c(331,331); lines(x2,y2,lty=3,col='blue')
> points(1.35,331,col='blue',pch=19)
> text(1.82,331,'Right Handed',col='blue')
>
```



```
> # Naive fixed effects analysis
> anova(lm(rtime ~ handed*ear))
Analysis of Variance Table

Response: rtime
             Df Sum Sq Mean Sq F value  Pr(>F)
handed        1    930  929.63  1.8556 0.17582
ear           2   2551 1275.41  2.5458 0.08286 .
handed:ear    2    615  307.41  0.6136 0.54317
Residuals   114  57113  500.99
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
>
> # Repeated measures with a mixed model
> dichotic = lmer(rtime ~ handed*ear + (1 | subject))
> Anova(dichotic, test="F") # F tests (from car package)
Analysis of Deviance Table (Type II Wald F tests with Kenward-Roger df)

Response: rtime
                F Df Df.res  Pr(>F)
handed     0.9706  1     38 0.33075
ear        4.6787  2     76 0.01213 *
handed:ear 1.1277  2     76 0.32914
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
>
> # Multiple comparisons: Which marginal means are different?
>
> # The combination variable HandEar will have 6 values
> n = length(rtime); n
[1] 120
> HandEar = character(n) # A character-valued variable of length n
> for(j in 1:n) HandEar[j] = paste(handed[j],ear[j],sep='')
> HandEar = factor(HandEar) # Maybe would be interpreted as a factor anyway
> head(data.frame(handed,ear,HandEar))
  handed   ear   HandEar
1   Left  Left  LeftLeft
2   Left Right LeftRight
3   Left  Both  LeftBoth
4   Left  Left  LeftLeft
5   Left Right LeftRight
6   Left  Both  LeftBoth
> table(HandEar) # Sample sizes
HandEar
  LeftBoth   LeftLeft  LeftRight  RightBoth  RightLeft RightRight
        20         20         20         20         20         20
>
> # Want a table of means in a similar format.
> meantable # Again
    Ear Handed Mean RT
1  Both   Left  317.90
2  Left   Left  326.60
3 Right   Left  332.80
4  Both  Right  315.10
5  Left  Right  324.65
6 Right  Right  320.85
> ComboMeans = meantable[,3] # All the rows, 3d column
> names(ComboMeans) = sort(unique(HandEar)); ComboMeans
  LeftBoth   LeftLeft  LeftRight  RightBoth  RightLeft RightRight
    317.90     326.60     332.80     315.10     324.65     320.85
```

```
>
> # For a no-intercept model on a combination variable, the regression
> # coefficients are the treatment combination means.
>
> # Fit a no-intercept model
> ComboModel = lmer(rtime ~ 0 + HandEar + (1 | subject))
>
> # Contrast matrix for testing Ear, just as a check
> CM1 = rbind(c(1, -1,  0,  1, -1,  0), # Both - Left
+             c(0,  1, -1,  0,  1, -1)) # Left - Right
> colnames(CM1) = sort(unique(HandEar)) # Makes it easier to look at
> CM1
     LeftBoth LeftLeft LeftRight RightBoth RightLeft RightRight
[1,]        1       -1         0         1        -1          0
[2,]        0        1        -1         0         1         -1
> linearHypothesis(ComboModel,CM1,test="F") # Compare F = 4.6787
Linear hypothesis test

Hypothesis:
HandEarLeftBoth - HandEarLeftLeft  + HandEarRightBoth - HandEarRightLeft = 0
HandEarLeftLeft - HandEarLeftRight  + HandEarRightLeft - HandEarRightRight = 0

Model 1: restricted model
Model 2: rtime ~ 0 + HandEar + (1 | subject)

  Res.Df Df      F  Pr(>F)
1     78
2     76  2 4.6787 0.01213 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
>
> # Now 3 Bonferroni corrected pairwise comparisons of marginal means for ear.
> # 0.05 divided by the number of tests =  0.05/3 = 0.01666667
>
> bothVSleft = c(1, -1,  0,  1, -1,  0)
> linearHypothesis(ComboModel,bothVSleft,test="F")
Linear hypothesis test

Hypothesis:
HandEarLeftBoth - HandEarLeftLeft  + HandEarRightBoth - HandEarRightLeft = 0

Model 1: restricted model
Model 2: rtime ~ 0 + HandEar + (1 | subject)

  Res.Df Df     F  Pr(>F)
1     77
2     76  1 6.109 0.01569 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
> bothVSright = c(1,  0, -1,  1,  0, -1)
> linearHypothesis(ComboModel,bothVSright,test="F")
Linear hypothesis test

Hypothesis:
HandEarLeftBoth - HandEarLeftRight  + HandEarRightBoth - HandEarRightRight = 0

Model 1: restricted model
Model 2: rtime ~ 0 + HandEar + (1 | subject)

  Res.Df Df      F   Pr(>F)
1     77
2     76  1 7.8214 0.006536 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
> leftVSright = c(0,  1, -1,  0,  1, -1)
> linearHypothesis(ComboModel,leftVSright,test="F")
Linear hypothesis test

Hypothesis:
HandEarLeftLeft - HandEarLeftRight  + HandEarRightLeft - HandEarRightRight = 0

Model 1: restricted model
Model 2: rtime ~ 0 + HandEar + (1 | subject)

  Res.Df Df      F Pr(>F)
1     77
2     76  1 0.1056  0.746
```

The Dichotic Listening data are balanced, and tests from `lmer` (with the `car` package) are the classical exact Fs. These do not exist for unbalanced designs.

Unbalanced Data: Ana's Reaction Time Data

English and Spanish speaking children reacted to sentences of three types (Exclamation, Question, Statement) at three levels of context (Full, Low, None). The response variable is median reaction time in 50 trials. Each subject contributes 9 medians.

```
> rm(list=ls()); options(scipen=999) # To avoid scientific notation
> # Install packages if necessary. Only need to do this once.
> # install.packages("lme4")
> # install.packages("car")
> # Load packages -- do this every time
> library(lme4) # For lmer function
Loading required package: Matrix
> library(car)  # For F-tests, likelihood ratio and Wald chi-squared tests
> # Avoid the lmerTest package. It seems to have actual errors.
>
> # Read data into a data frame
> rt = read.table("anaRT2.data.txt") # Edited to be more unbalanced than it really
is.
> head(rt,12) # First 12 lines
   PARTICIPANT Language sentencetype context   rtime
1          E01  English     Question    None  2955.0
2          E01  English  Exclamation    None  2877.0
3          E01  English    Statement    None  2978.5
4          E01  English     Question     Low  3758.0
5          E01  English  Exclamation     Low  5393.0
6          E01  English    Statement     Low  3380.5
7          E01  English     Question    Full 21058.5
8          E01  English  Exclamation    Full 20875.0
9          E01  English    Statement    Full 17702.5
10         E02  English     Question    None  3271.0
11         E02  English  Exclamation    None  3134.0
12         E02  English    Statement    None  3898.5
> # Notice Full context.
> summary(rt)
  PARTICIPANT     Language        sentencetype context        rtime
 E01     :  9   English:135   Exclamation:90   Full:90   Min.   : 2254
 E02     :  9   Spanish:135   Question   :90   Low :90   1st Qu.: 3004
 E03     :  9                 Statement  :90   None:90   Median : 3679
 E04     :  9                                            Mean   : 8783
 E05     :  9                                            3rd Qu.:18221
 E06     :  9                                            Max.   :22734
 (Other):216                                             NA's   :10
> rt = na.omit(rt) # Now it's unbalanced
> rt = subset(rt,context!='Full') # Full context gives MUCH slower reaction times
> attach(rt)
> table(sentencetype, context)
             context
sentencetype  Full Low None
  Exclamation    0  24   30
  Question       0  29   30
  Statement      0  30   30
> context=factor(context)
> table(context, sentencetype)
       sentencetype
context Exclamation Question Statement
   Low           24       29        30
   None          30       30        30
```

```
>
> Ns = table(context, sentencetype,  Language); Ns
, , Language = English

       sentencetype
context Exclamation Question Statement
   Low            9       14        15
   None          15       15        15

, , Language = Spanish

       sentencetype
context Exclamation Question Statement
   Low           15       15        15
   None          15       15        15

>
> meanz = aggregate(rtime, by=list(context,sentencetype,Language), FUN=mean)
> meanz
   Group.1     Group.2 Group.3        x
1      Low Exclamation English 3031.944
2     None Exclamation English 3013.833
3      Low    Question English 3073.964
4     None    Question English 3517.800
5      Low   Statement English 2840.833
6     None   Statement English 3632.833
7      Low Exclamation Spanish 3366.133
8     None Exclamation Spanish 3586.867
9      Low    Question Spanish 2955.700
10    None    Question Spanish 3745.933
11     Low   Statement Spanish 3264.233
12    None   Statement Spanish 3553.133
> meanz = meanz[,4]; dim(meanz) = c(2,3,2)
> meanz
, , 1

         [,1]     [,2]     [,3]
[1,] 3031.944 3073.964 2840.833
[2,] 3013.833 3517.800 3632.833

, , 2

         [,1]     [,2]     [,3]
[1,] 3366.133 2955.700 3264.233
[2,] 3586.867 3745.933 3553.133

>
> # Use nice labels from Ns
> meantable = Ns
> meantable[1,,1] = meanz[1,,1]
> meantable[2,,1] = meanz[2,,1]
> meantable[1,,2] = meanz[1,,2]
> meantable[2,,2] = meanz[2,,2]
```

```
> meantable
, , Language = English

       sentencetype
context Exclamation Question Statement
   Low     3031.944 3073.964  2840.833
   None    3013.833 3517.800  3632.833

, , Language = Spanish

       sentencetype
context Exclamation Question Statement
   Low     3366.133 2955.700  3264.233
   None    3586.867 3745.933  3553.133

>
> # For unbalanced designs, use effect coding (contr.sum) on all factors and
> # use the type="III" (Type III sums of squares) option on the Anova function.
> # This way, tests of main effects and interactions will correspond to what you
> # would get by testing contrasts on a combination variable, which is usually
> # what you want.
>
> contrasts(Language) = contr.sum
> contrasts(context) = contr.sum
> contrasts(sentencetype) = contr.sum
> contrasts(sentencetype)
            [,1] [,2]
Exclamation    1    0
Question       0    1
Statement     -1   -1
>
> reaction1 = lmer(rtime ~ Language*sentencetype*context + (1 | PARTICIPANT) )
> Anova(reaction1, test="F", type="III")
Analysis of Deviance Table (Type III Wald F tests with Kenward-Roger df)

Response: rtime
                                     F Df  Df.res                Pr(>F)
(Intercept)                  2682.3734  1  28.329 < 0.00000000000000022 ***
Language                        2.9058  1  28.329              0.099206 .
sentencetype                    0.1935  2 134.152              0.824288
context                        29.6945  1 133.951          0.0000002349 ***
Language:sentencetype           1.7813  2 134.152              0.172373
Language:context                0.0937  1 133.951              0.760009
sentencetype:context            5.1662  2 134.152              0.006896 **
Language:sentencetype:context   3.4458  2 134.152              0.034729 *
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

```
>
> # Marginal table of Sentence Type by Context
> scon = apply(meantable,c(1,2),mean); scon
       sentencetype
context Exclamation Question Statement
   Low     3199.039 3014.832  3052.533
   None    3300.350 3631.867  3592.983
> # Difference between differences
> scon[2,] - scon[1,] # None minus Low
Exclamation    Question   Statement
   101.3111    617.0345    540.4500


>
> # What about the 3-way?
> meantable[2,,1] - meantable[1,,1] # None-Low for English
Exclamation    Question   Statement
  -18.11111   443.83571   792.00000
> meantable[2,,2] - meantable[1,,2] # None-Low for Spanish
Exclamation    Question   Statement
   220.7333    790.2333    288.9000
>
```

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http://www.utstat.toronto.edu/~brunner/oldclass/appliedf18