STA 442/2101 Formulas

$M_y(t) = E(e^{yt})$ $\qquad$ $M_{ay}(t) = M_y(at)$

$M_{y+a}(t) = e^{at}M_y(t)$ $\qquad$ $M_{\sum_{i=1}^n y_i}(t) = \prod_{i=1}^n M_{y_i}(t)$

$y \sim N(\mu,\sigma^2)$ means $M_y(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$ $\qquad$ $y \sim \chi^2(\nu)$ means $M_y(t) = (1-2t)^{-\nu/2}$

If $W = W_1 + W_2$ with W_1 and W_2 independent, $W \sim \chi^2(\nu_1+\nu_2)$, $W_2 \sim \chi^2(\nu_2)$ then $W_1 \sim \chi^2(\nu_1)$

Columns of $\mathbf{A}$ *linearly dependent* means there is a vector $\mathbf{v} \neq \mathbf{0}$ with $\mathbf{Av} = \mathbf{0}$.

Columns of $\mathbf{A}$ *linearly independent* means that $\mathbf{Av} = \mathbf{0}$ implies $\mathbf{v} = \mathbf{0}$.

$\mathbf{A}$ *positive definite* means $\mathbf{v}^\top \mathbf{Av} > 0$ for all vectors $\mathbf{v} \neq \mathbf{0}$.

$\boldsymbol{\Sigma} = \mathbf{P}\boldsymbol{\Lambda}\mathbf{P}^\top$ $\qquad$ $\boldsymbol{\Sigma}^{-1} = \mathbf{P}\boldsymbol{\Lambda}^{-1}\mathbf{P}^\top$

$\boldsymbol{\Sigma}^{1/2} = \mathbf{P}\boldsymbol{\Lambda}^{1/2}\mathbf{P}^\top$ $\qquad$ $\boldsymbol{\Sigma}^{-1/2} = \mathbf{P}\boldsymbol{\Lambda}^{-1/2}\mathbf{P}^\top$

$cov(\mathbf{w}) = E\left\{(\mathbf{w}-\boldsymbol{\mu}_w)(\mathbf{w}-\boldsymbol{\mu}_w)^\top\right\}$ $\qquad$ $cov(\mathbf{w},\mathbf{t}) = E\left\{(\mathbf{w}-\boldsymbol{\mu}_w)(\mathbf{t}-\boldsymbol{\mu}_t)^\top\right\}$

$cov(\mathbf{w}) = E\{\mathbf{w}\mathbf{w}^\top\} - \boldsymbol{\mu}_w\boldsymbol{\mu}_w^\top$ $\qquad$ $cov(\mathbf{Aw}) = \mathbf{A}cov(\mathbf{w})\mathbf{A}^\top$

If $\mathbf{w} \sim N_p(\boldsymbol{\mu},\boldsymbol{\Sigma})$, then $\mathbf{Aw}+\mathbf{c} \sim N_r(\mathbf{A}\boldsymbol{\mu}+\mathbf{c}, \mathbf{A}\boldsymbol{\Sigma}\mathbf{A}^\top)$ and $(\mathbf{w}-\boldsymbol{\mu})^\top\boldsymbol{\Sigma}^{-1}(\mathbf{w}-\boldsymbol{\mu}) \sim \chi^2(p)$

$L(\boldsymbol{\mu},\boldsymbol{\Sigma}) = |\boldsymbol{\Sigma}|^{-n/2}(2\pi)^{-np/2}\exp -\frac{n}{2}\left\{tr(\widehat{\boldsymbol{\Sigma}}\boldsymbol{\Sigma}^{-1}) + (\overline{\mathbf{y}}-\boldsymbol{\mu})^\top\boldsymbol{\Sigma}^{-1}(\overline{\mathbf{y}}-\boldsymbol{\mu})\right\}$, where $\widehat{\boldsymbol{\Sigma}} = \frac{1}{n}\sum_{i=1}^n(\mathbf{y}_i-\overline{\mathbf{y}})(\mathbf{y}_i-\overline{\mathbf{y}})^\top$

$y_i = \beta_0 + \beta_1 x_i + \epsilon_i$ $\qquad$ $\widehat{\beta}_0 = \overline{y} - \widehat{\beta}_1\overline{x}$

$\widehat{\beta}_1 = \frac{\sum_{i=1}^n(x_i-\overline{x})(y_i-\overline{y})}{\sum_{i=1}^n(x_i-\overline{x})^2} = \frac{\sum_{i=1}^n x_iy_i - n\,\overline{x}\,\overline{y}}{\sum_{i=1}^n x_i^2 - n\overline{x}^2}$ $\qquad$ $r = \frac{\sum_{i=1}^n(x_i-\overline{x})(y_i-\overline{y})}{\sqrt{\sum_{i=1}^n(x_i-\overline{x})^2}\sqrt{\sum_{i=1}^n(y_i-\overline{y})^2}}$

$y_i = \beta_0 + \beta_1 x_{i,1} + \cdots + \beta_{p-1}x_{i,p-1} + \epsilon_i$ $\qquad$ $\epsilon_1,\ldots,\epsilon_n$ independent $N(0,\sigma^2)$

$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$ $\qquad$ $\boldsymbol{\epsilon} \sim N_n(\mathbf{0},\sigma^2\mathbf{I}_n)$

$\widehat{\boldsymbol{\beta}} = (\mathbf{X}^\top\mathbf{X})^{-1}\mathbf{X}^\top\mathbf{y} \sim N_p\left(\boldsymbol{\beta},\sigma^2(\mathbf{X}^\top\mathbf{X})^{-1}\right)$ $\qquad$ $\widehat{\mathbf{y}} = \mathbf{X}\widehat{\boldsymbol{\beta}} = \mathbf{Hy}$, where $\mathbf{H} = \mathbf{X}(\mathbf{X}^\top\mathbf{X})^{-1}\mathbf{X}^\top$

$\mathbf{e} = \mathbf{y} - \widehat{\mathbf{y}} = (\mathbf{I}-\mathbf{H})\mathbf{y}, \quad \mathbf{X}^\top\mathbf{e} = \mathbf{0}$ $\qquad$ $\widehat{\boldsymbol{\beta}}$ and $\mathbf{e}$ are independent under normality.

$\sum_{i=1}^n(y_i-\overline{y})^2 = \sum_{i=1}^n(y_i-\widehat{y}_i)^2 + \sum_{i=1}^n(\widehat{y}_i-\overline{y})^2$ $\qquad$ $SST = SSE + SSR$ and $R^2 = \frac{SSR}{SST}$

$\frac{SSE}{\sigma^2} = \frac{\mathbf{e}^\top\mathbf{e}}{\sigma^2} \sim \chi^2(n-p)$ $\qquad$ $MSE = \frac{SSE}{n-p}$

$T = \frac{Z}{\sqrt{W/\nu}} \sim t(\nu)$ $\qquad$ $F = \frac{W_1/\nu_1}{W_2/\nu_2} \sim F(\nu_1,\nu_2)$

Under $H_0: \mathbf{L}\boldsymbol{\beta} = \mathbf{h}$, $F^* = \frac{(\mathbf{L}\widehat{\boldsymbol{\beta}}-\mathbf{h})^\top(\mathbf{L}(\mathbf{X}^\top\mathbf{X})^{-1}\mathbf{L}^\top)^{-1}(\mathbf{L}\widehat{\boldsymbol{\beta}}-\mathbf{h})}{r\,MSE} = \frac{SSR_F - SSR_R}{r\,MSE_F} \sim F(r,n-p)$

If $\lim_{n\to\infty} E(T_n) = \theta$ and $\lim_{n\to\infty} Var(T_n) = 0$, then $T_n \stackrel{p}{\rightarrow} \theta$

If $\sqrt{n}(T_n - \mu) \stackrel{d}{\rightarrow} T \sim N(0, \sigma^2)$, then $\sqrt{n}\left(g(T_n) - g(\mu)\right) \stackrel{d}{\rightarrow} g'(\mu)T \sim N(0, g'(\mu)^2\sigma^2)$

If $\mathbf{T}_n \stackrel{d}{\rightarrow} \mathbf{T}$ and $\mathbf{Y}_n \stackrel{p}{\rightarrow} \mathbf{c}$, then $\left(\begin{array}{c} \mathbf{T}_n \\ \\ \mathbf{Y}_n \end{array}\right) \stackrel{d}{\rightarrow} \left(\begin{array}{c} \mathbf{T} \\ \\ \mathbf{c} \end{array}\right)$ $\qquad \sqrt{n}(\overline{\mathbf{x}}_n - \boldsymbol{\mu}) \stackrel{d}{\rightarrow} \mathbf{x} \sim N(\mathbf{0}, \boldsymbol{\Sigma})$

Let $g : \mathbb{R}^d \rightarrow \mathbb{R}^k$ etc. If $\sqrt{n}(\mathbf{T}_n - \boldsymbol{\theta}) \stackrel{d}{\rightarrow} \mathbf{T}$, then $\sqrt{n}(g(\mathbf{T}_n) - g(\boldsymbol{\theta})) \stackrel{d}{\rightarrow} \dot{\mathrm{g}}(\boldsymbol{\theta})\mathbf{T}$, where $\dot{\mathrm{g}}(\boldsymbol{\theta}) = \left[\frac{\partial g_i}{\partial \theta_j}\right]_{k\times d}$

$G^2 = -2\log\left(\frac{\max_{\theta\in\Theta_0} L(\theta)}{\max_{\theta\in\Theta} L(\theta)}\right) = -2\log\left(\frac{L(\widehat{\theta}_0)}{L(\widehat{\theta})}\right)$ $\qquad W_n = (\mathbf{L}\widehat{\boldsymbol{\theta}}_n - \mathbf{h})^\top \left(\mathbf{L}\widehat{\mathbf{V}}_n\mathbf{L}^\top\right)^{-1} (\mathbf{L}\widehat{\boldsymbol{\theta}}_n - \mathbf{h})$

$\log\left(\frac{\pi_i}{1-\pi_i}\right) = \beta_0 + \beta_1 x_{i,1} + \ldots + \beta_{p-1}x_{i,p-1}$ $\qquad \pi_i = \frac{e^{\beta_0 + \beta_1 x_{i,1} + \ldots + \beta_{p-1}x_{i,p-1}}}{1+e^{\beta_0 + \beta_1 x_{i,1} + \ldots + \beta_{p-1}x_{i,p-1}}}$

$\log(\lambda_i) = \beta_0 + \beta_1 x_{i,1} + \ldots + \beta_{p-1}x_{i,p-1}$

$$\begin{aligned} \log\left(\frac{\pi_1}{\pi_3}\right) &= \beta_{0,1} + \beta_{1,1}x_1 + \ldots + \beta_{p-1,1}x_{p-1} = L_1 \\ \log\left(\frac{\pi_2}{\pi_3}\right) &= \beta_{0,2} + \beta_{1,2}x_1 + \ldots + \beta_{p-1,2}x_{p-1} = L_2 \end{aligned}$$

$$\begin{aligned} \pi_1 &= \frac{e^{L_1}}{1+e^{L_1}+e^{L_2}} \\ \pi_2 &= \frac{e^{L_2}}{1+e^{L_1}+e^{L_2}} \\ \pi_3 &= \frac{1}{1+e^{L_1}+e^{L_2}} \end{aligned}$$

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> df = 1:12
> Critical_Value = qchisq(0.95,df)
> cbind(df,Critical_Value)
      df Critical_Value
 [1,]  1       3.841459
 [2,]  2       5.991465
 [3,]  3       7.814728
 [4,]  4       9.487729
 [5,]  5      11.070498
 [6,]  6      12.591587
 [7,]  7      14.067140
 [8,]  8      15.507313
 [9,]  9      16.918978
[10,] 10      18.307038
[11,] 11      19.675138
[12,] 12      21.026070
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