STA 442/2101 Formulas

$M_Y(t) = E(e^{Yt})$

$M_{aY}(t) = M_Y(at)$

$M_{Y+a}(t) = e^{at}M_Y(t)$

$M_{\sum_{i=1}^n Y_i}(t) = \prod_{i=1}^n M_{Y_i}(t)$

$Y \sim N(\mu,\sigma^2)$ means $M_Y(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$

$Y \sim \chi^2(\nu)$ means $M_Y(t) = (1-2t)^{-\nu/2}$

If $W = W_1 + W_2$ with W_1 and W_2 independent, $W \sim \chi^2(\nu_1+\nu_2)$, $W_2 \sim \chi^2(\nu_2)$ then $W_1 \sim \chi^2(\nu_1)$

Columns of $\mathbf{A}$ *linearly dependent* means there is a vector $\mathbf{v} \neq \mathbf{0}$ with $\mathbf{Av} = \mathbf{0}$.

Columns of $\mathbf{A}$ *linearly independent* means that $\mathbf{Av} = \mathbf{0}$ implies $\mathbf{v} = \mathbf{0}$.

$\mathbf{A}$ *positive definite* means $\mathbf{v}^\top \mathbf{Av} > 0$ for all vectors $\mathbf{v} \neq \mathbf{0}$.

$\boldsymbol{\Sigma} = \mathbf{P}\boldsymbol{\Lambda}\mathbf{P}^\top$

$\boldsymbol{\Sigma}^{-1} = \mathbf{P}\boldsymbol{\Lambda}^{-1}\mathbf{P}^\top$

$\boldsymbol{\Sigma}^{1/2} = \mathbf{P}\boldsymbol{\Lambda}^{1/2}\mathbf{P}^\top$

$\boldsymbol{\Sigma}^{-1/2} = \mathbf{P}\boldsymbol{\Lambda}^{-1/2}\mathbf{P}^\top$

$cov(\mathbf{w}) = E\left\{(\mathbf{w}-\boldsymbol{\mu}_w)(\mathbf{w}-\boldsymbol{\mu}_w)^\top\right\}$

$C(\mathbf{w},\mathbf{t}) = E\left\{(\mathbf{w}-\boldsymbol{\mu}_w)(\mathbf{t}-\boldsymbol{\mu}_t)^\top\right\}$

$cov(\mathbf{w}) = E\{\mathbf{w}\mathbf{w}^\top\} - \boldsymbol{\mu}_w\boldsymbol{\mu}_w^\top$

$cov(\mathbf{Aw}) = \mathbf{A}cov(\mathbf{w})\mathbf{A}^\top$

If $\mathbf{w} \sim N_p(\boldsymbol{\mu},\boldsymbol{\Sigma})$, then $\mathbf{Aw}+\mathbf{c} \sim N_r(\mathbf{A}\boldsymbol{\mu}+\mathbf{c}, \mathbf{A}\boldsymbol{\Sigma}\mathbf{A}^\top)$

and $(\mathbf{w}-\boldsymbol{\mu})^\top\boldsymbol{\Sigma}^{-1}(\mathbf{w}-\boldsymbol{\mu}) \sim \chi^2(p)$

$Y_i = \beta_0 + \beta_1 x_{i,1} + \cdots + \beta_{p-1}x_{i,p-1} + \epsilon_i$

$\epsilon_1,\ldots,\epsilon_n$ independent $N(0,\sigma^2)$

$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$

$\boldsymbol{\epsilon} \sim N_n(\mathbf{0},\sigma^2\mathbf{I}_n)$

$\widehat{\boldsymbol{\beta}} = (\mathbf{X}^\top\mathbf{X})^{-1}\mathbf{X}^\top\mathbf{y} \sim N_p\left(\boldsymbol{\beta}, \sigma^2(\mathbf{X}^\top\mathbf{X})^{-1}\right)$

$\widehat{\mathbf{y}} = \mathbf{X}\widehat{\boldsymbol{\beta}} = \mathbf{Hy}$, where $\mathbf{H} = \mathbf{X}(\mathbf{X}^\top\mathbf{X})^{-1}\mathbf{X}^\top$

$\mathbf{e} = \mathbf{y} - \widehat{\mathbf{y}} = (\mathbf{I}-\mathbf{H})\mathbf{y}$

$\widehat{\boldsymbol{\beta}}$ and $\mathbf{e}$ are independent under normality.

$\frac{SSE}{\sigma^2} = \frac{\mathbf{e}^\top\mathbf{e}}{\sigma^2} \sim \chi^2(n-p)$

$MSE = \frac{SSE}{n-p}$

$\sum_{i=1}^n (Y_i-\overline{Y})^2 = \sum_{i=1}^n (Y_i-\widehat{Y}_i)^2 + \sum_{i=1}^n (\widehat{Y}_i-\overline{Y})^2$

$SST = SSE + SSR$ and $R^2 = \frac{SSR}{SST}$

$T = \frac{Z}{\sqrt{W/\nu}} \sim t(\nu)$

$F = \frac{W_1/\nu_1}{W_2/\nu_2} \sim F(\nu_1,\nu_2)$

$T = \frac{\mathbf{a}^\top\widehat{\boldsymbol{\beta}} - \mathbf{a}^\top\boldsymbol{\beta}}{\sqrt{MSE\,\mathbf{a}^\top(\mathbf{X}^\top\mathbf{X})^{-1}\mathbf{a}}} \sim t(n-p)$

$T = \frac{Y_{n+1} - \mathbf{x}_{n+1}^\top\widehat{\boldsymbol{\beta}}}{\sqrt{MSE\,(1+\mathbf{x}_{n+1}^\top(\mathbf{X}^\top\mathbf{X})^{-1}\mathbf{x}_{n+1})}} \sim t(n-p)$

$F^* = \frac{SSR_F - SSR_R}{r\,MSE_F} \sim F(r,n-p)$

$a = \frac{R_F^2 - R_R^2}{1-R_R^2} = \frac{rF^*}{n-p+rF^*}$

$F^* = \frac{(\mathbf{L}\widehat{\boldsymbol{\beta}}-\mathbf{h})^\top(\mathbf{L}(\mathbf{X}^\top\mathbf{X})^{-1}\mathbf{L}^\top)^{-1}(\mathbf{L}\widehat{\boldsymbol{\beta}}-\mathbf{h})}{r\,MSE} \sim F(r,n-p)$

$F^* = \left(\frac{n-p}{r}\right)\left(\frac{a}{1-a}\right) \sim F(r,n-p)$

$r = \frac{\sum_{i=1}^n (x_i-\overline{x})(y_i-\overline{y})}{\sqrt{\sum_{i=1}^n (x_i-\overline{x})^2}\sqrt{\sum_{i=1}^n (y_i-\overline{y})^2}}$

If $\sqrt{n}\,(T_n-\theta) \stackrel{d}{\rightarrow} T \sim N(0,\sigma^2)$ then $\sqrt{n}\,(g(T_n)-g(\theta)) \stackrel{d}{\rightarrow} Y \sim N\left(0, g^\prime(\theta)^2\,\sigma^2\right)$.

$Z_i \stackrel{ind}{\sim} N(\mu_i,1) \Rightarrow \sum_{i=1}^n Z_i^2 \sim \chi^2_{nc}(n, \lambda = \sum_{i=1}^n \mu_i^2)$

$\lambda = \frac{(\mathbf{L}\boldsymbol{\beta}-\mathbf{h})^\top(\mathbf{L}(\mathbf{X}^\top\mathbf{X})^{-1}\mathbf{L}^\top)^{-1}(\mathbf{L}\boldsymbol{\beta}-\mathbf{h})}{\sigma^2}$