Multivariate Testing

In studies with both between and within-cases factors

Monkey Study

- Train monkeys on 5 different discrimination tasks, at 16, 12, 8, 4 and 2 weeks prior to treatment
- Order of tasks different for each monkey.
- Treatment is to block function of the hippocampus (with drug, not surgery) , re-tested on all 5 tasks. Get 5 scores.
- 11 randomly assigned to treatment, 7 to control
- Treatment is between, time elapsed since training is within

Form linear combinations

- Four difference variables for the five time levels
- Also, calculate average score across the 5 time levels
- Now we have 5 potential response variables

Main effect of Time

- Response variables are the difference variables
- With effect coding, intercepts are grand means - Averages of the between cases cell means
- Test of all 4 intercepts = zero is a test of whether, averaging across the between subjects factor, the marginal means of the within subjects factor are equal
- This is the test for the main effect of time.

Main effect of Treatment

- Response variable is average discrimination score, averaging across time
- Means of this variable for the two treatment groups are the marginal means, averaging across time
- So just do an ordinary *F* test for difference between means
- This is testing the main effect of Treatment.

Interaction

- DV is the set of difference variables
- IV is Treatment
- If the set of mean differences (trend over time) *depends* on treatment, then there is an interaction

In general, with both within and between-cases factors

- Suppose there are *k* measurement for each case
- Calculate contrast variables to represent main effects and interactions of the within-cases factors. There will be *k*-1 of them.
- Calculate one more variable: The mean or sum of the *k* measurements

Tests

- Tests of within-cases main effects and interactions are tests on the intercepts
- Tests of between-cases main effects and interactions are tests on the sum or average variable
- Interactions between between and within-cases factors: Tests of the between-cases factors on the contrast variables
- For example, if A and B are between and C and D are within, test of AxB on the contrasts representing the CxD interaction is: the AxBxCxD interaction, because the CxD interaction depends on the AxB combination.

Multivariate test statistics

- Wilks' Lambda
- Pillai's Trace
- Hotelling-Lawley Trace
- Roy's Greatest Root

The four multivariate test statistics

- All control Type I error properly
- Differ somewhat in power, sometimes, but none is most powerful all the time
- Distributions under H_0 are known
  - Tables of critical values are available
  - Exact p-values are nasty to compute
  - There are F *approximations*, sometimes exact

I like Wilks' Lambda

- F approximations are best (p-values are more often exactly right)
- Based most directly on the likelihood ratio, so I understand it most easily
- Scheffé tests are relatively easy to construct
- With no explanatory variables, intercepts are just expected values, and Hotelling's T-squared is

$$T^2 = (n-1)\left(\frac{1}{\lambda} - 1\right)$$

But there are problems

- Same explanatory variables for all response variables: No time-varying covariates
- Sometimes there are lots of measurements on each subject, and a very big covariance matrix to estimate.
- If any data are missing, the whole case must be discarded.