Name Jerry

Student Number

STA 431 Quiz 4

1. (5 points) Suppose you estimate the $m \times 1$ parameter vector $\boldsymbol{\theta}$ by numerical maximum likelihood, obtaining the vector of MLEs $\widehat{\boldsymbol{\theta}}_n$ and the estimated asymptotic covariance matrix $\widehat{\mathbf{V}}_n$. Give a z statistic for testing $H_0 : \mathbf{a}^\top \boldsymbol{\theta} = 0$, where $\mathbf{a}$ is an $m \times 1$ non-zero vector of constants. Start with the asymptotic distribution of $\mathbf{a}^\top \widehat{\boldsymbol{\theta}}_n$. Your final answer is a formula for z. **Circle the formula.**

$a^T \hat{\theta}_n \stackrel{\cdot}{\sim} N(a^T\theta, a^T V_n a)$, so

$$\frac{a^T\hat{\theta}_n - a^T\theta}{\sqrt{a^T V_n a}} \stackrel{\cdot}{\sim} N(0,1)$$, and the

test statistic for $H_0 : a^T\theta = 0$ is

$$z = \frac{a^T\hat{\theta}_n}{\sqrt{a^T \hat{V}_n a}}$$

2. (2 points) Starting with your answer to Question 1, show that for $H_0 : \mathbf{a}^\top \boldsymbol{\theta} = h$, the Wald test statistic $W_n = z^2$.

$$W_n = (a^T \hat{\theta}_n - 0)^T (a^T \hat{V}_n a)^{-1} (a^T \hat{\theta}_n - 0)$$

$$= \frac{(a^T \hat{\theta}_n)^2}{a^T \hat{V}_n a}$$ Because all 3 matrices are 1×1

$$= z^2$$

3. (3 points) In Question 3 of this week's assignment, you calculated a 95% confidence interval for the quantity $2\alpha - \beta$, where α and β were the parameters of a beta distribution. Your answer was a set of two numbers, the lower confidence limit and the upper confidence limit. *Write the numbers in the space below.*

$(-0.395, 0.166)$

On your printout, circle the numbers and write "Question 3" beside them. *Do not answer this question if you do not have a printout.*

Please attach your printout to the quiz paper. The printout should show your *complete* R input and output. Make sure your name and student number appear on the printout.

R work for Question 3

```
> # Q3: Numerical MLE for beta
>
> rm(list=ls())
> bdata = scan("https://www.utstat.toronto.edu/brunner/openSEM/data/beta24.data.txt")
Read 500 items
> # Beta minus log likelihood
> bmll = function(ab,xx)
+     {
+     nn = length(xx); a = ab[1]; b = ab[2]
+     value = nn*(lgamma(a)+lgamma(b) - lgamma(a+b)) -
+             (a-1)*sum(log(xx)) - (b-1)*sum(log((1-xx)))
+     return(value)
+     } # End of function bmll
>
> bsearch = optim(par=c(1,1), fn = bmll,
+                 method = "L-BFGS-B", lower = c(0,0), hessian=TRUE, xx=bdata)
> bsearch

$par
[1] 1.956054 4.026869

$value
[1] -184.783

$counts
function gradient
      12       12

$convergence
[1] 0

$message
[1] "CONVERGENCE: REL_REDUCTION_OF_F <= FACTR*EPSMCH"

$hessian
          [,1]      [,2]
[1,] 240.64932 -90.94232
[2,] -90.94232  49.90191
```

```
> # (a) MLE
> thetahat = bsearch$par
> thetahat # (alphahat, betahat)
[1] 1.956054 4.026869

> # (b) Likelihood ratio test of H0: beta = 2 alpha
> # Search restricted parameter space
> bmll0 = function(beta,datta) bmll(c(beta/2,beta), xx=datta)
> bsearch0 = optim(par=1, fn = bmll0, method = "L-BFGS-B", lower = 0, datta=bdata)
> bsearch0

$par
[1] 3.943671

$value
[1] -184.458

$counts
function gradient
       9        9

$convergence
[1] 0

$message
[1] "CONVERGENCE: REL_REDUCTION_OF_F <= FACTR*EPSMCH"

>
> # (c) Test H0: mu = 2.1. Reject if |z| > 1.96
> z = (muhat-2.1)/se_muhat; z
[1] -2.844819

> Gsq = 2 * (bsearch0$value - bsearch$value)
> dfree=1
> pval = 1-pchisq(Gsq,dfree)
> c(Gsq,dfree,pval)
[1] 0.6498473 1.0000000 0.4201673

>
> # (c) Wald test
> source("https://www.utstat.toronto.edu/brunner/openSEM/fun/Wtest.txt")
> L = rbind(c(2,-1))
> Vhat = solve(bsearch$hessian)
> Wtest(L,Tn=thetahat,Vn=Vhat)
        W        df   p-value
0.6436909 1.0000000 0.4223774

>
> # (d) CI for 2 alpha - beta
> a = rbind(2,-1)
> se = as.numeric(sqrt(t(a)%*%Vhat%*%a)); se
[1] 0.1430393

> est = as.numeric(t(a)%*%thetahat); est
[1] -0.114761

> # (est/se)^2 # Wald stat
> lower95 = est - 1.96*se; upper95 = est + 1.96*se
> c(lower95, upper95)
[1] -0.3951180  0.1655961
```

Question 3