STA 431 Formulas

$Pr\{Y \in A\} = \sum_x Pr\{Y \in A | X = x\} p_x(x)$ $\quad$ $Pr\{Y \in A\} = \int_{-\infty}^{\infty} Pr\{X \in A | X = x\} f_x(x)\, dx$

$Var(X) \stackrel{def}{=} E\left((X - \mu_x)^2 \right)$ $\quad$ $Var(X) = E(X^2) - [E(X)]^2$

$Cov(X,Y) \stackrel{def}{=} E\left((X - \mu_x)(Y - \mu_{_Y}) \right)$ $\quad$ $Cov(X,Y) = E(XY) - E(X)E(Y)$

$Cov(aX, bY) = ab\, Cov(X,Y)$ $\quad$ $Cov(x + a, Y + b) = Cov(X,Y)$

$Corr(X,Y) \stackrel{def}{=} \frac{Cov(X,Y)}{\sqrt{Var(X)Var(Y)}}$ $\quad$ $Cov\left(\sum_{i=1}^n a_i X_i \, , \, \sum_{j=1}^m b_j Y_j \right) = \sum_{i=1}^n \sum_{j=1}^m a_i b_j Cov\left(X_i, Y_j\right)$

$\mathbf{Av} = \lambda \mathbf{v}$ $\quad$ $\mathbf{A} = \mathbf{CDC}^\top$ with $\mathbf{CC}^\top = \mathbf{C}^\top\mathbf{C} = \mathbf{I}$, for $\mathbf{A}$ symmetric.

$\mathbf{A}^{-1} = \mathbf{CD}^{-1}\mathbf{C}^\top$

$\mathbf{A}^{1/2} = \mathbf{CD}^{1/2}\mathbf{C}^\top$ $\quad$ $\mathbf{A}^{-1/2} = \mathbf{CD}^{-1/2}\mathbf{C}^\top$

$\mathbf{A}$ *positive definite* means $\mathbf{v}'\mathbf{A}\mathbf{v} > 0$ for all vectors $\mathbf{v} \neq \mathbf{0}$.

$cov(\mathbf{x}) \stackrel{def}{=} E\left\{ (\mathbf{x} - \boldsymbol{\mu}_x)(\mathbf{x} - \boldsymbol{\mu}_x)^\top \right\}$ $\quad$ $cov(\mathbf{x}, \mathbf{y}) \stackrel{def}{=} E\left\{ (\mathbf{x} - \boldsymbol{\mu}_x)(\mathbf{y} - \boldsymbol{\mu}_y)^\top \right\}$

$cov(\mathbf{Ax}) = \mathbf{A}\, cov(\mathbf{x})\, \mathbf{A}^\top$ $\quad$ $cov(\mathbf{Ax}, \mathbf{By}) = \mathbf{A}\, cov(\mathbf{x}, \mathbf{y})\, \mathbf{B}^\top$

$\mathbf{L} = \mathbf{A}_1\mathbf{X}_1 + \cdots + \mathbf{A}_m\mathbf{X}_m + \mathbf{b}$ $\quad$ $cov(\mathbf{L}_1, \mathbf{L}_2) = \sum_{i=1}^m \sum_{j=1}^n \mathbf{A}_i\, cov(\mathbf{x}_i, \mathbf{y}_j)\, \mathbf{B}_j^\top$

If $\mathbf{x} \sim N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, then $\mathbf{Ax} + \mathbf{b} \sim N_r(\mathbf{A}\boldsymbol{\mu} + \mathbf{b}, \mathbf{A}\boldsymbol{\Sigma}\mathbf{A}^\top)$ and $(\mathbf{x} - \boldsymbol{\mu})^\top \boldsymbol{\Sigma}^{-1}(\mathbf{x} - \boldsymbol{\mu}) \sim \chi^2(p)$

$L(\boldsymbol{\mu}, \boldsymbol{\Sigma}) = |\boldsymbol{\Sigma}|^{-n/2}(2\pi)^{-np/2} \exp -\frac{n}{2}\left\{ tr(\widehat{\boldsymbol{\Sigma}}\boldsymbol{\Sigma}^{-1}) + (\overline{\mathbf{x}} - \boldsymbol{\mu})^\top \boldsymbol{\Sigma}^{-1}(\overline{\mathbf{x}} - \boldsymbol{\mu}) \right\}$

$\widehat{\boldsymbol{\mu}} = \overline{\mathbf{x}}, \quad \widehat{\boldsymbol{\Sigma}} = \frac{1}{n}\sum_{i=1}^n (\mathbf{x}_i - \overline{\mathbf{x}})(\mathbf{x}_i - \overline{\mathbf{x}})^\top$ $\quad$ $\widehat{\sigma}_x^2 = \frac{1}{n}\sum_{i=1}^n (x_i - \overline{x})^2, \quad \widehat{\sigma}_{xy} = \frac{1}{n}\sum_{i=1}^n (x_i - \overline{x})(y_i - \overline{y})$

If $\mathbf{t}_n \stackrel{p}{\rightarrow} \mathbf{c}$ and $g(\mathbf{x})$ is continuous at $\mathbf{x} = \mathbf{c}$, then $g(\mathbf{t}_n) \stackrel{p}{\rightarrow} g(\mathbf{c})$.

If $\mathbf{x}_1, \ldots, \mathbf{x}_n \stackrel{iid}{\sim} (\boldsymbol{\mu}, \boldsymbol{\Sigma})$, then $\overline{\mathbf{x}}_n \stackrel{p}{\rightarrow} \boldsymbol{\mu}$ and $\overline{\mathbf{x}}_n \stackrel{.}{\sim} N_p(\boldsymbol{\mu}, \frac{1}{n}\boldsymbol{\Sigma})$

$\widehat{\boldsymbol{\theta}}_n \stackrel{p}{\rightarrow} \boldsymbol{\theta}$ and $\widehat{\boldsymbol{\theta}}_n \stackrel{.}{\sim} N_m(\boldsymbol{\theta}, \mathbf{V}_n)$, with $\widehat{\mathbf{V}}_n = \mathbf{H}^{-1}$, where $\mathbf{H} = \left[\frac{\partial^2(-\ell)}{\partial\theta_i \partial\theta_j} \right]_{\boldsymbol{\theta} = \widehat{\boldsymbol{\theta}}}$

$G^2 = -2\ln\left(\frac{\max_{\theta \in \Theta_0} L(\theta)}{\max_{\theta \in \Theta} L(\theta)} \right) = -2\ln\left(\frac{L(\widehat{\theta}_0)}{L(\widehat{\theta})} \right) \sim \chi^2(r)$ if $H_0 : \theta \in \Theta_0$ is true.

$W_n = (\mathbf{L}\widehat{\boldsymbol{\theta}}_n - \mathbf{h})^\top \left(\mathbf{L}\widehat{\mathbf{V}}_n\mathbf{L}^\top \right)^{-1} (\mathbf{L}\widehat{\boldsymbol{\theta}}_n - \mathbf{h}) \stackrel{.}{\sim} \chi^2(r)$ if $H_0 : \mathbf{L}\boldsymbol{\theta} = \mathbf{h}$ is true.

The reliability of an observable variable d as a measurement of a latent variable F is $\rho^2 \stackrel{def}{=} (Corr(F,d))^2$.

If $d = F + e$ with $Var(F) = \phi$ and $Var(e) = \omega$, then $\rho^2 = \frac{\phi}{\phi + \omega}$.

For symmetric $\boldsymbol{\Sigma}_{k \times k}$, there are $k(k+1)/2$ unique elements and $k(k-1)/2$ unique off-diagonal elements.

The Double Measurement Model in centered form:

$$\mathbf{y}_i = \boldsymbol{\beta}\mathbf{x}_i + \boldsymbol{\epsilon}_i \qquad cov(\mathbf{x}_i) = \boldsymbol{\Phi}_x,\; cov(\boldsymbol{\epsilon}_i) = \boldsymbol{\Psi}$$

$$\mathbf{F}_i = \left(\begin{array}{c} \mathbf{x}_i \\ \hline \mathbf{y}_i \end{array} \right) \qquad \mathbf{x}_i \text{ is } p \times 1,\; \mathbf{y}_i \text{ is } q \times 1,\; \mathbf{F}_i \text{ is } (p+q) \times 1 \qquad cov(\mathbf{F}_i) = \boldsymbol{\Phi}$$

$$\mathbf{d}_{i,1} = \mathbf{F}_i + \mathbf{e}_{i,1} \qquad cov(\mathbf{e}_{i,1}) = \boldsymbol{\Omega}_1,\; cov(\mathbf{e}_{i,2}) = \boldsymbol{\Omega}_2$$

$$\mathbf{d}_{i,2} = \mathbf{F}_i + \mathbf{e}_{i,2} \qquad \mathbf{x}_i,\; \boldsymbol{\epsilon}_i,\; \mathbf{e}_{i,1} \text{ and } \mathbf{e}_{i,2} \text{ are independent.}$$

The General Structural Equation Model in centered form:

$$\mathbf{y}_i = \boldsymbol{\beta}\mathbf{y}_i + \boldsymbol{\Gamma}\mathbf{x}_i + \boldsymbol{\epsilon}_i \qquad cov(\mathbf{x}_i) = \boldsymbol{\Phi}_x \text{ and } cov(\boldsymbol{\epsilon}_i) = \boldsymbol{\Psi}$$

$$\mathbf{F}_i = \left(\begin{array}{c} \mathbf{x}_i \\ \hline \mathbf{y}_i \end{array} \right) \qquad cov(\mathbf{F}_i) = \boldsymbol{\Phi} = \left(\begin{array}{c|c} \boldsymbol{\Phi}_{11} & \boldsymbol{\Phi}_{12} \\ \hline \boldsymbol{\Phi}_{12}^\top & \boldsymbol{\Phi}_{22} \end{array} \right)$$

$$\mathbf{d}_i = \boldsymbol{\Lambda}\mathbf{F}_i + \mathbf{e}_i \qquad cov(\mathbf{e}_i) = \boldsymbol{\Omega}$$

$\mathbf{x}_i$, $\boldsymbol{\epsilon}_i$ and $\mathbf{e}_i$ are independent. $\mathbf{x}_i$ is $p \times 1$, $\mathbf{y}_i$ is $q \times 1$, $\mathbf{d}_i$ is $k \times 1$.

$\boldsymbol{\Phi}_x$ and $\boldsymbol{\Psi}$ are positive definite.

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> df = 1:8; CriticalValue = qchisq(0.95,df)
> round(rbind(df,CriticalValue),3)

               [,1]  [,2]  [,3]  [,4]  [,5]   [,6]   [,7]   [,8]
df            1.000 2.000 3.000 4.000  5.00  6.000  7.000  8.000
CriticalValue 3.841 5.991 7.815 9.488 11.07 12.592 14.067 15.507
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