Name ______________________________

Student Number ______________________________

# STA 312 f2023 Quiz 1

Let the random variable $X$ have an exponential distribution (see formula sheet on reverse), and let $Y = a\,X$, where the constant $a > 0$. Derive the probability density function of $Y$. Show your work. Do not forget to indicate where the density is non-zero.

# STA 312f23 Formulas

| **Distribution** | **Density or probability mass function** | **Expected value** | **Variance** |
|---|---|---|---|
| Bernoulli($\theta$) | $p(x) = \theta^x(1-\theta)^{1-x}$ for $x = 0, 1$ | $\theta$ | $\theta(1-\theta)$ |
| Binomial($n, \theta$) | $p(k) = \binom{n}{k}\theta^k(1-\theta)^{n-k}$ for $k = 0, 1, \ldots, n$ | $n\theta$ | $n\theta(1-\theta)$ |
| Geometric($\theta$) | $p(k) = (1-\theta)^{k-1}\,\theta$ for $k = 1, 2, \ldots$ | $1/\theta$ | $(1-\theta)/\theta^2$ |
| Poisson($\lambda$) | $p(k) = \frac{e^{-\lambda}\,\lambda^k}{k!}$ for $k = 0, 1, \ldots$ | $\lambda$ | $\lambda$ |
| Exponential($\lambda$) | $f(x) = \lambda e^{-\lambda x}$ for $x \geq 0$ | $1/\lambda$ | $1/\lambda^2$ |
| Gamma($\alpha, \lambda$) | $f(x) = \frac{\lambda^\alpha}{\Gamma(\alpha)} e^{-\lambda x}\, x^{\alpha-1}$ for $x \geq 0$ | $\alpha/\lambda$ | $\alpha/\lambda^2$ |
| Normal($\mu, \sigma^2$) | $f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp - \left\{\frac{(x-\mu)^2}{2\sigma^2}\right\}$ | $\mu$ | $\sigma^2$ |
| Chi-squared($\nu$) | $f(x) = \frac{1}{2^{\frac{\nu}{2}}\Gamma(\frac{\nu}{2})} e^{-\frac{x}{2}}\, x^{\frac{\nu}{2}-1}$ for $x \geq 0$ | $\nu$ | $2\nu$ |

$E(X) \stackrel{def}{=} \sum_x x\, p_x(x)$ or $\int_{-\infty}^{\infty} x\, f_x(x)\, dx$

$E(g(X)) = \sum_x g(x)\, p_x(x)$ or $\int_{-\infty}^{\infty} g(x)\, f_x(x)\, dx$

$Var(X) \stackrel{def}{=} E\left((X-\mu)^2\right)$

$Var(X) = E(X^2) - [E(X)]^2$

If $X \sim N(\mu, \sigma^2)$, then $\frac{X-\mu}{\sigma} \sim N(0,1)$.

If $Z \sim N(0,1)$, then $Z^2 \sim \chi^2(1)$.