## STA 302 Formulas

Columns of **A** *linearly dependent* means there is a vector  $\mathbf{v} \neq \mathbf{0}$  with  $\mathbf{A}\mathbf{v} = \mathbf{0}$ .

Columns of A *linearly independent* means that Av = 0 implies v = 0.

$$\begin{split} \boldsymbol{\Sigma} &= \mathbf{C}\mathbf{D}\mathbf{C}' \qquad \boldsymbol{\Sigma}^{-1} &= \mathbf{C}\mathbf{D}^{-1}\mathbf{C}' \\ \boldsymbol{\Sigma}^{1/2} &= \mathbf{C}\mathbf{D}^{1/2}\mathbf{C}' \qquad \boldsymbol{\Sigma}^{-1/2} &= \mathbf{C}\mathbf{D}^{-1/2}\mathbf{C}' \\ M_Y(t) &= E(e^{Yt}) \qquad M_{aY}(t) &= M_Y(at) \\ M_{Y+a}(t) &= e^{at}M_Y(t) \qquad M_{\sum_{i=1}^n Y_i}(t) &= \prod_{i=1}^n M_{Y_i}(t) \\ Y &\sim N(\mu, \sigma^2) \text{ means } M_Y(t) &= e^{\mu t + \frac{1}{2}\sigma^2 t^2} \qquad Y &\sim \chi^2(\nu) \text{ means } M_Y(t) &= (1 - 2t)^{-\nu/2} \\ \text{If } W &= W_1 + W_2 \text{ with } W_1 \text{ and } W_2 \text{ independent, } W &\sim \chi^2(\nu_1 + \nu_2), W_2 &\sim \chi^2(\nu_2) \text{ then } W_1 &\sim \chi^2(\nu_1) \\ \text{cov}(\mathbf{Y}) &= E\left\{(\mathbf{Y} - \mu_y)(\mathbf{Y} - \mu_y)'\right\} \qquad C(\mathbf{Y}, \mathbf{T}) &= E\left\{(\mathbf{Y} - \mu_y)(\mathbf{T} - \mu_t)'\right\} \\ \text{cov}(\mathbf{Y}) &= E\{\mathbf{Y}\mathbf{Y}'\} - \mu_y\mu'_y \qquad \text{cov}(\mathbf{A}\mathbf{Y}) &= \mathbf{A}\text{cov}(\mathbf{Y})\mathbf{A}' \\ M_{\mathbf{Y}}(t) &= E(e^{t'\mathbf{Y}}) \qquad M_{\mathbf{A}\mathbf{Y}}(t) &= M_{\mathbf{Y}}(\mathbf{A}'t) \\ M_{\mathbf{Y}+e}(t) &= e^{t'e}M_{\mathbf{Y}}(t) \qquad \mathbf{Y} &\sim N_p(\mu, \mathbf{\Sigma}) \text{ means } M_{\mathbf{Y}}(t) &= e^{t'\mu + \frac{1}{2}t'\mathbf{\Sigma}t} \\ \mathbf{Y}_1 \text{ and } \mathbf{Y}_2 \text{ are independent if and only if } M_{(\mathbf{Y}_1,\mathbf{Y}_2)'} \left((\mathbf{t}_1,\mathbf{t}_2)'\right) &= M_{\mathbf{Y}_1}(\mathbf{t}_1)M_{\mathbf{Y}_2}(\mathbf{t}_2) \end{split}$$

$$\begin{split} & \text{If } \mathbf{Y} \sim N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma}), \text{ then } \mathbf{AY} \sim N_q(\mathbf{A}\boldsymbol{\mu}, \mathbf{A}\boldsymbol{\Sigma}\mathbf{A}'), & \text{and } W = (\mathbf{Y} - \boldsymbol{\mu})'\boldsymbol{\Sigma}^{-1}(\mathbf{Y} - \boldsymbol{\mu}) \sim \chi^2(p) \\ & r_{xy} = \frac{\sum_{i=1}^n (X_i - \overline{X})(Y_i - \overline{Y})}{\sqrt{\sum_{i=1}^n (X_i - \overline{X})^2}\sqrt{\sum_{i=1}^n (Y_i - \overline{Y})^2}} \\ & Y_i = \beta_0 + \beta_1 x_{i1} + \dots + \beta_k x_{ik} + \epsilon_i & \epsilon_1, \dots, \epsilon_n \text{ independent } N(0, \sigma^2) \\ & \mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon} & \boldsymbol{\epsilon} \sim N_n(\mathbf{0}, \sigma^2 \mathbf{I}_n) \\ & \hat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y} & \hat{\mathbf{Y}} = \mathbf{X}\hat{\boldsymbol{\beta}} \\ & \sum_{i=1}^n (Y_i - \overline{Y})^2 = \sum_{i=1}^n (Y_i - \hat{Y}_i)^2 + \sum_{i=1}^n (\hat{Y}_i - \overline{Y})^2 & SST = SSE + SSR \text{ and } R^2 = \frac{SSR}{SST} \\ & \hat{\boldsymbol{\epsilon}} = \mathbf{Y} - \hat{\mathbf{Y}} & \hat{\boldsymbol{\beta}} \sim N_{k+1} \left(\boldsymbol{\beta}, \sigma^2(\mathbf{X}'\mathbf{X})^{-1}\right) \\ & \hat{\boldsymbol{\beta}} \text{ and } \hat{\boldsymbol{\epsilon}} \text{ are independent under normality.} & SSE/\sigma^2 = \hat{\boldsymbol{\epsilon}}'\hat{\boldsymbol{c}}/\sigma^2 \sim \chi^2(n-k-1) \\ & T = \frac{Z}{\sqrt{W/\nu}} \sim t(\nu) & F = \frac{W_1/\nu_1}{W_2/\nu_2} \sim F(\nu_1, \nu_2) \\ & T = \frac{a'\hat{\boldsymbol{\beta}} - a'\boldsymbol{\beta}}{s\sqrt{a'(\mathbf{X'X})^{-1}\mathbf{a}}} \sim t(n-k-1) & T = \frac{Y_0 - \mathbf{x}'_0\hat{\boldsymbol{\beta}}}{qs^2} \sim t(n-k-1), \text{ where } s^2 = MSE = \frac{SSE}{n-k-1} \end{aligned}$$

This formula sheet was prepared by Jerry Brunner, Department of Statistics, University of Toronto. It is licensed under a Creative Commons Attribution - ShareAlike 3.0 Unported License. Use any part of it as you like and share the result freely. The LATEX source code is available from the course website: http://www.utstat.toronto.edu/~brunner/oldclass/302f13