Name Jerry

Student Number ____________________

STA 302s13 Quiz 2A

1. (2 points) Recall that the moment-generating function of a random variable X is defined by $M_X(t) = E(e^{Xt})$. Let $Y = aX$, where a is a constant. Find the moment-generating function of Y. Show your work. **Circle your answer.**

$$M_Y(t) = M_{aX}(t) = E\left(e^{(aX)t}\right) = E\left(e^{X(at)}\right) = M_X(at)$$

2. (2 points) Let X be a random variable with moment-generating function $M_X(t)$, and let $Y = X + b$, where b is a constant. Find the moment-generating function of Y. Show your work. **Circle your answer.**

$$M_Y(t) = M_{X+b}(t) = E\left(e^{(X+b)t}\right) = E\left(e^{Xt+bt}\right)$$

$$= E\left(e^{Xt}e^{bt}\right) = e^{bt}E(e^{Xt}) = e^{bt}M_X(t)$$

3. (2 points) Recall that if $X \sim N(\mu, \sigma^2)$, it has moment-generating function $M_X(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$. Let $X \sim N(\mu, \sigma^2)$ and $Y = aX + b$, where a and b are constants. Find the distribution of Y. Show your work.

$$M_Y(t) = M_{aX+b}(t) = e^{bt}M_{aX}(t) = e^{bt}M_X(at)$$

$$= e^{bt}e^{\mu(at)+\frac{1}{2}\sigma^2(at)^2} = e^{(a\mu+b)t+\frac{1}{2}(a^2\sigma^2)t^2}$$

So $Y \sim N(a\mu+b, a^2\sigma^2)$

4. (4 points) Recall that a Chi-squared random variable X with parameter $\nu > 0$ has moment-generating function $M_X(t) = (1 - 2t)^{-\nu/2}$. Let $Y = X_1 + X_2$, where X_1 and X_2 are independent, $X_1 \sim \chi^2(\nu_1)$ and $Y \sim \chi^2(\nu_1 + \nu_2)$, where ν_1 and ν_2 are both positive. What is the distribution of X_2? Show your work.

$$M_Y(t) = M_{X_1 + X_2}(t) \overset{\text{independence}}{=} M_{X_1}(t)\, M_{X_2}(t)$$

$$(1-2t)^{-\frac{(\nu_1+\nu_2)}{2}} \qquad (1-2t)^{-\nu_1/2} M_{X_2}(t)$$

$$\Rightarrow \cancel{(1-2t)^{-\nu_1/2}}\,(1-2t)^{-\nu_2/2} = \cancel{(1-2t)^{-\nu_1/2}}\, M_{X_2}(t)$$

$$\Rightarrow M_{X_2}(t) = (1-2t)^{-\nu_2/2}$$

So $X_2 \sim \chi^2(\nu_2)$

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STA 302s13 Quiz 2B

Please be careful here! This question is a bit like Question 1 of the homework, but it is not the same. If you answer the homework question instead of what is asked here, you will get a zero.

In the homework problem, the intercept was fixed at zero. Here, the slope is fixed at one. Accordingly, let $Y_i = \beta_0 + x_i + \epsilon_i$ for $i = 1, \ldots, n$, where $\epsilon_1, \ldots, \epsilon_n$ are a random sample from a distribution with expected value zero and variance σ^2, and β_0 and σ^2 are unknown constants. The independent variable values $x_1, \ldots, x_n$ are fixed, observable constants.

1. (2 points) What is $E(Y_i)$? Show some work. **Circle your answer**.

$E(Y_i) = E(\beta_0 + x_i + \varepsilon_i) = \beta_0 + x_i + E(\varepsilon_i) = \beta_0 + x_i$

2. (5 points) Find the Least Squares estimate of β_0 by minimizing the function

$$Q(\beta_0) = \sum_{i=1}^{n} (Y_i - \beta_0 - x_i)^2$$

over all values of β_0. Let $\widehat{\beta}_0$ denote the point at which $Q(\beta_0)$ is minimal. Show your work. **Circle your expression for** $\widehat{\beta}_0$. You need not bother with a second derivative test.

$$\frac{dQ}{d\beta_0} = \sum_{i=1}^{n} \frac{d}{d\beta_0} (Y_i - \beta_0 - x_i)^2 = \sum_{i=1}^{n} 2(Y_i - \beta_0 - x_i)(-1) \overset{set}{=} 0$$

$$\Rightarrow 0 = \sum_{i=1}^{n} (Y_i - \beta_0 - x_i) = \sum_{i=1}^{n} Y_i - n\beta_0 - \sum_{i=1}^{n} x_i$$

$$\Rightarrow n\beta_0 = \sum_{i=1}^{n} Y_i - \sum_{i=1}^{n} x_i \Rightarrow \beta_0 = \frac{1}{n}\sum_{i=1}^{n} Y_i - \frac{1}{n}\sum_{i=1}^{n} x_i$$

So

$$\hat{\beta}_0 = \overline{Y} - \overline{x}$$

3. (3 points) Recall that a statistic is an *unbiased estimator* of a parameter if the expected value of the statistic is equal to the parameter. Is $\widehat{\beta}_0$ an unbiased estimator of β? Answer Yes or No. **Circle the word Yes or the word No**. Prove your answer and show your work.

Yes

$$E(\hat{\beta}_0) = E(\bar{Y} - \bar{x}) = E(\bar{Y}) - \bar{x}$$

$$= E\left(\frac{1}{n}\sum_{i=1}^{n} Y_i\right) - \bar{x} = \frac{1}{n}\sum_{i=1}^{n} E(Y_i) - \bar{x}$$

$$= \frac{1}{n}\sum_{i=1}^{n} (\beta_0 + x_i) - \bar{x} = \cancel{\frac{1}{n}}\,\cancel{n}\,\beta_0 + \cancel{\frac{1}{n}\sum_{i=1}^{n} x_i - \bar{x}}$$

$$= \beta_0$$