Sample Questions: Moment-generating functions

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1. Let X have a moment-generating function $M_X(t)$ and let a be a constant. Show $M_{aX}(t) = M_X(at)$.

2. Let X have a moment-generating function $M_X(t)$ and let a be a constant. Show $M_{a+X}(t) = e^{at} M_X(t)$.

3. Let X and Y be independent, (continuous) random variables. Show $M_{X+Y}(t) = M_X(t)\, M_Y(t)$.

4. Let $X \sim N(0,1)$. Calculate $M_X(t)$.

5. Let $X \sim N(\mu, \sigma^2)$. Calculate $M_X(t)$.

6. Let $X \sim N(\mu, \sigma^2)$. Show $Y = \frac{X-\mu}{\sigma} \sim N(0,1)$ using moment-generating functions.

7. Let $X \sim N(\mu, \sigma^2)$. Find the distribution of $Y = a + bX$ using moment-generating functions.

8. Let $X_1 \sim N(\mu_1, \sigma_1^2)$ and $X_2 \sim N(\mu_2, \sigma_2^2)$ be independent. Find the distribution of $Y = aX_1 + bX_2 + c$.

9. Let $Z \sim N(0,1)$ and let $Y = Z^2$. Find the distribution of Y. Note that the MGF of a chi-squared random variable is $M(t) = (1-2t)^{-\frac{\nu}{2}}$.

10. Independently for $i = 1, \ldots, n$, let $Y_i \sim \chi^2(\nu_i)$. Find the distribution of $W = \sum_{i=1}^n Y_i$.

11. Independently for $i = 1, \ldots, n$, let $X_i \sim N(\mu_i, \sigma_i)$. What is the distribution of $Y = \sum_{i=1}^n \left(\frac{X_i - \mu_i}{\sigma_i}\right)^2$? Justify your answer.