Sets[1]

STA 256: Fall 2019

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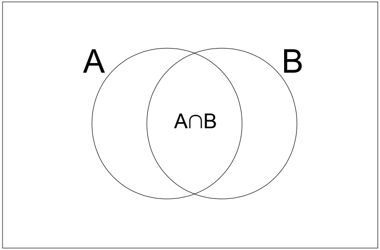
Statistical Experiment
This vocabulary is not in the text

A statistical experiment is a procedure whose outcome is not known in advance with certainty.

Sample Space: set of outcomes $s \in S$

- Sell 500 lottery tickets, pick the winning number.
  $S = \{1, 2, \ldots, 500\}$
- Hold your breath as long as you can.
  $S = \{t : t \geq 0\}$
- Pick coin or die from jar, roll or toss.
  $S = \{H, T, 1, 2, 3, 4, 5, 6\}$

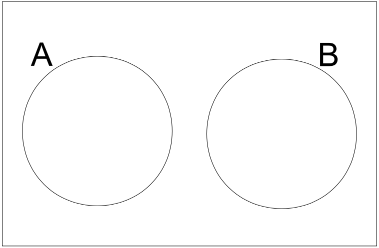
Event: Set of outcomes, $A \subset S$



- $A \cap B = \{s \in S : s \in A \text{ and } s \in B\}$
- $A \cup B = \{s \in S : s \in A \text{ or } s \in B\}$
- $A^c = \{s \in S : s \notin A\}$
- $A \cap B^c = \{s \in S : s \in A \text{ and } s \notin B\}$

Disjoint sets

- A and B are said to be *disjoint* if $A \cap B = \emptyset$
- The idea is that A and B have no elements in common; they do not overlap.



- However, recall that the null set is a subset of every set: $\emptyset \subseteq A$.
- So $\emptyset \cap A = \emptyset$.
- And the null set is also disjoint from every set.

Set Laws

No proofs, just Venn diagrams at most

- Commutative: $A \cup B = B \cup A$, $A \cap B = B \cap A$
- Associative
  - $(A \cup B) \cup C = A \cup (B \cup C)$,
  - $(A \cap B) \cap C = A \cap (B \cap C)$
- Distributive (like multiplication)
  - $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
  - $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

De Morgan Laws

Not in the text

- $(A \cap B)^c = A^c \cup B^c$
- $(A \cup B)^c = A^c \cap B^c$
- Rule: complement and flip $\cup\cap$

Extend the notation to larger number of sets

Not in the text

Distributive laws

- $A \cap \left(\cup_{j=1}^{n} B_j\right) = \cup_{j=1}^{n}(A \cap B_j)$, or even
- $A \cap \left(\cup_{j=1}^{\infty} B_j\right) = \cup_{j=1}^{\infty}(A \cap B_j)$

  and
- $A \cup \left(\cap_{j=1}^{n} B_j\right) = \cap_{j=1}^{n}(A \cup B_j)$
- $A \cup \left(\cap_{j=1}^{\infty} B_j\right) = \cap_{j=1}^{\infty}(A \cup B_j)$

De Morgan Laws (complement and flip)

- $(\cap_{j=1}^{\infty} A_j)^c = \cup_{j=1}^{\infty} A_j^c$
- $(\cup_{j=1}^{\infty} A_j)^c = \cap_{j=1}^{\infty} A_j^c$

WARNING!

- Addition and subtraction apply only to numbers. They are not set operations.
- For example, if A and B are sets, then $A + B$ is not defined.
- $A + B$ is not the same as $A \cup B$.
- Because what is $A + A - A$?

- Some very bad probability proofs use addition and subtraction of sets.
- Such proofs will receive a zero.

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