Sample Questions: Joint Distributions Part One

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1. The discrete random variables x and y have joint probability mass function $p_{_{X,Y}} = cxy$ for $x = 1, 2, 3$, $y = 1, 2$, and zero otherwise.

   (a) Find the value of the constant c and calculate the marginal probability functions.

   (b) What is $F_{_X}(x)$?

2. The discrete random variables x and y have joint distribution

	$x = 1$	$x = 2$	$x = 3$
$y = 2$	$3/12$	$1/12$	$3/12$
$y = 1$	$1/12$	$3/12$	$1/12$

Give the following. The answers are numbers.

(a) $F_{X,Y}(1, 1)$ $F_{X,Y}(2, 2)$

(b) $F_{X,Y}(1.5, 4)$ $F_{X,Y}(-1, 3)$

(c) $F_{X,Y}(4, 4)$ $F_{X,Y}(6, 1.82)$

(d) $F_{X,Y}(4, 19)$ $F_{X,Y}(0, 0)$

	$x = 1$	$x = 2$	$x = 3$
$y = 2$	$3/12$	$1/12$	$3/12$
$y = 1$	$1/12$	$3/12$	$1/12$

3. A jar contains 30 red marbles, 50 green marbles and 20 blue marbles. A sample of 15 marbles is selected *with replacement*. Let X be the number of red marbles and Y be the number of blue marbles. What is the joint probability mass function of X and Y?

$p(x, y) =$

4. This time the selection is without replacement. Again, what is the joint probability mass function of X and Y?

$p(x, y) =$

5. Let $f_{X,Y}(x,y) = \begin{cases} c(x+y) & \text{for } 0 \le x \le 1 \text{ and } 0 \le y \le x \\ 0 & \text{otherwise} \end{cases}$

(a) Find the constant c.

(b) What is $f_X(x)$?

6. The continuous random variables X and Y have joint cumulative distribution function

$$F_{X,Y}(x,y) = \begin{cases} x^3 - x^3 e^{-y/4} & \text{for } 0 \le x \le 1 \text{ and } y \ge 0 \\ 1 - e^{-y/4} & \text{for } x > 1 \text{ and } y \ge 0 \\ 0 & \text{otherwise} \end{cases}$$

(a) What is $F_{X,Y}(\frac{1}{2}, 3)$?

(b) What is $F_{X,Y}(2, 3)$?

(c) What is $F_{X,Y}(-1, 3)$?

(d) What is $f_{X,Y}(x, y)$?

7. Still for the joint distribution with

$$F_{X,Y}(x,y) = \begin{cases} x^3 - x^3 e^{-y/4} & \text{for } 0 \le x \le 1 \text{ and } y \ge 0 \\ 1 - e^{-y/4} & \text{for } x > 1 \text{ and } y \ge 0 \\ 0 & \text{otherwise} \end{cases} \quad \text{and}$$

$$f_{X,Y}(x,y) = \begin{cases} 3x^2 \frac{1}{4} e^{-y/4} & \text{for } 0 \le x \le 1 \text{ and } y \ge 0 \\ 0 & \text{otherwise} \end{cases}$$

(a) Obtain $f_X(x)$ by integrating out y.

(b) Calculate $F_X(x)$ by taking limits.

(c) Obtain $f_X(x)$ from $F_X(x)$.

For $F_{X,Y}(x,y) = \begin{cases} x^3 - x^3 e^{-y/4} & \text{for } 0 \le x \le 1 \text{ and } y \ge 0 \\ 1 - e^{-y/4} & \text{for } x > 1 \text{ and } y \ge 0 \\ 0 & \text{otherwise} \end{cases}$ and

$$f_{X,Y}(x,y) = \begin{cases} 3x^2 \frac{1}{4} e^{-y/4} & \text{for } 0 \le x \le 1 \text{ and } y \ge 0 \\ 0 & \text{otherwise} \end{cases}$$

(d) Obtain $f_Y(y)$ by integrating out x.

(e) Obtain $F_Y(y)$ by taking limits.

(f) Obtain $f_Y(y)$ from $F_Y(y)$.

8. Let $f_{X,Y}(x,y) = \begin{cases} 2e^{-(x+y)} & \text{for } 0 \leq x \leq y \text{ and } y \geq 0 \\ 0 & \text{otherwise} \end{cases}$

Obtain $f_X(x)$.

9. Let $f_{X,Y}(x,y) = \begin{cases} \frac{xy}{16} & \text{for } 0 \leq x \leq 2 \text{ and } 0 \leq y \leq 4 \\ 0 & \text{otherwise} \end{cases}$

Find $P(Y < X^2)$. The answer is a number.

10. Let $f_{X,Y}(x,y) = \left\{ \begin{array}{ll} 4xy\, e^{-(x^2+y^2)} & \mbox{for } x \geq 0 \mbox{ and } y \geq 0 \\ 0 & \mbox{otherwise} \end{array} \right.$

Find $P(Y > X)$. The answer is a number.

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http://www.utstat.toronto.edu/~brunner/oldclass/256f19