Sample Questions: Independence

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1. A jar contains 5 red balls and 15 black balls. Draw 2 balls randomly with replacement.

   (a) What is the probability that the first ball is red and the second is black? The answer is a number.

   (b) What is the probability of one red and one black in any order? The answer is a number.

2. Let A and B be events of positive probability. Is it possible for A and B to be both disjoint and independent? Answer Yes or No and prove your answer.

3. Roll a fair die n times.

   (a) What is the probability of observing at least one 4?

   (b) How many times must you roll the die for the probability of at least one 4 to be 0.90 or more? The answer is a number.

4. A biased coin has $P(\text{Head}) = p$. Toss it three times.

   (a) List the elements of the sample space, along with their probabilities.

   (b) What is $P(\text{Two Heads})$?

5. It is clear from the last problem that the probability of a string with k heads is the same, regardless of their placement. Suppose we toss the biased coin n times. What is the probability of k heads (for $k = 0, \ldots, n$)?

6. Again, a biased coin has $P(\text{Head}) = p$. Toss it until the first head occurs, and then stop.

   (a) What is the probability that the first head appears on the fifth toss?

   (b) What is the probability that a head eventually occurs (on toss 1 or 2 or ...)?

(c) What is the probability that the first head occurs on an even numbered toss (toss 2 or 4 or ...)?

7. In repeated tosses of a coin with $P(\text{Head}) = \theta$, what is the probability that the third head comes on the seventh toss?