Sample Questions: Conditional Probability[1]

1. The table below shows percentages of passengers on the Titanic.

	Died	Lived	
1st Class	9	15	0.24
2nd Class	13	9	0.22
3d Class	40	14	0.54
	0.62	0.38	

For a randomly chosen passenger, what is

(a) The probability of living?

0.38

(b) The probability of living

i. Given 1st class?

$$P(L|1) = \frac{P(L \cap 1)}{P(1)} = \frac{0.15}{0.24} = 0.625$$

ii. Given 2nd class?

$$P(L|2) = \frac{P(L \cap 2)}{P(2)} = \frac{.09}{0.22} = 0.409$$

iii. Given 3d class?

$$P(L|3) = \frac{P(L \cap 3)}{P(3)} = \frac{0.14}{0.54} = 0.259$$

(c) The probability of being in first class given that the person died?

$$P(1|D) = \frac{P(1 \cap D)}{P(D)} = \frac{.09}{.62} = 0.145$$

[1]STA256 Fall 2019. Copyright information is at the end of the last page.

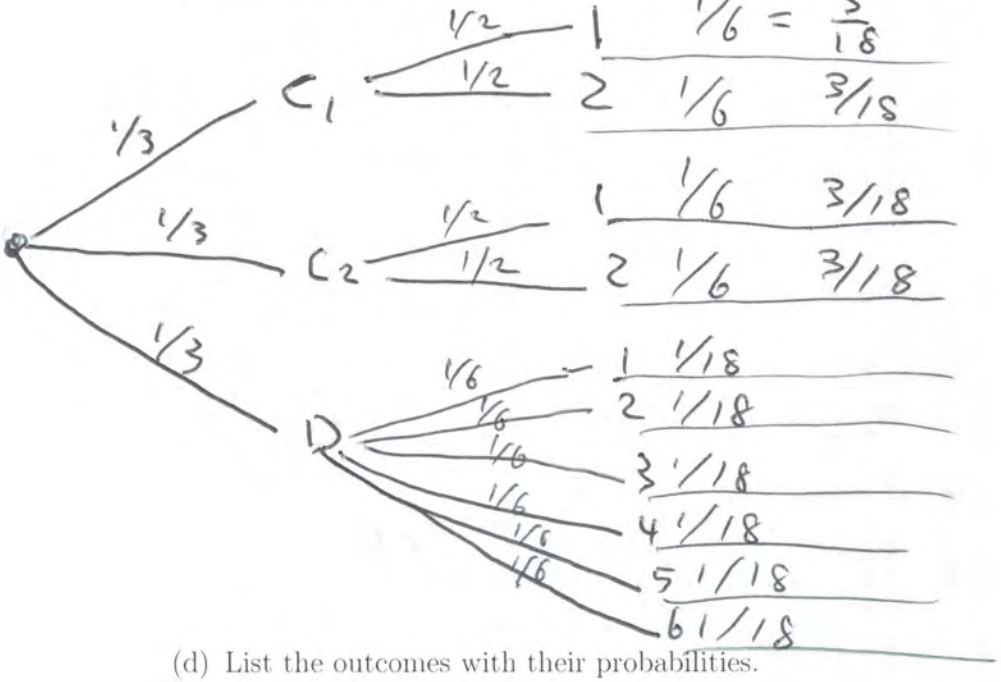
2. A jar contains two fair coins and one fair die. The coins have a "1" on one side and a "2" on the other side. Pick an object at random, roll or toss, and observe the number.

(a) What is $P(2 \cap C)$?

$= P(C)\,P(2|C) = \frac{2}{3} \cdot \frac{1}{2} = \boxed{\frac{1}{3}}$

(b) What is $P(6|C)$? $=0$ $\frac{P(C \cap 6)}{P(C)} = \frac{P(\emptyset)}{2/3} = \frac{0}{2/3} = 0$

(c) Make a tree diagram.



(d) List the outcomes with their probabilities.

1	2	3	4	5	6
3/18	3/18	1/18	1/18	1/18	1/18

(e) What is $P(C|2)$?

$\frac{P(C \cap 2)}{P(2)} = \frac{3/18 + 3/18}{7/18} = \boxed{\frac{6}{7}}$

3. Let $S = \cup_{k=1}^{\infty} A_k$, disjoint, with $P(A_k) > 0$ for all k. Using the formula sheet ~~and the tabular format illustrated in lecture~~, prove $P(B) = \sum_{k=1}^{\infty} P(B|A_k)P(A_k)$.

We have already seen

$$P(B) = \sum_{k=1}^{\infty} P(A_k \cap B)$$

$$= \sum_{k=1}^{\infty} P(A_k) P(B|A_k) \quad \blacksquare$$

4. Prove the following version of Bayes' Theorem. Let $S = \cup_{k=1}^{\infty} A_k$, disjoint, with $P(A_k) > 0$ for all k. Then

$$P(A_j|B) = \frac{P(B|A_j)P(A_j)}{\sum_{k=1}^{\infty} P(B|A_k)P(A_k)}.$$

You may use anything from the formula sheet except Bayes' theorem itself.

$$P(A_j|B) = \frac{P(A_j \cap B)}{P(B)} = \frac{P(A_j)P(B|A_j)}{P(B)}$$

$$= \frac{P(A_j)P(B|A_j)}{\sum_{k=1}^{\infty} P(A_k)P(B|A_k)} \quad \blacksquare$$

5. Two balls are drawn in succession from a jar containing three red balls and four white balls. What is the probability that the first ball was white given that the second ball was red? The answer is a number. Circle your answer.

$$P(W_1 \mid R_2) = \frac{P(W_1 \cap R_2)}{P(R_2)} = \frac{P(W_1)P(R_2 \mid W_1)}{P(R_2)}$$

$$= \frac{P(W_1)P(R_2 \mid W_1)}{P(W_1)P(R_2 \mid W_1) + P(R_1)P(R_2 \mid R_1)}$$

$$= \frac{\frac{4}{7} \cdot \frac{1}{2}}{\frac{2}{7} + \frac{3}{7} \cdot \frac{1}{3}} = \frac{2/7}{3/7}$$

$$= \frac{2}{3}$$

$P(D) = \frac{1}{1000}$

6. This is an important real-world application of Bayes' Theorem. Suppose only one person in a thousand has some rare disease. We have a screening test for the disease, and it's a good test.

   - 90% of those with the disease test positive. $P(T|D) = \frac{900}{1000}$
   - 95% of those without the disease test negative. $P(T|D^c) = \frac{50}{1000}$

   Given a positive test, what is the probability that the person actually has the disease? The answer is a number. Circle your answer.

$$P(D|T) = \frac{P(D \cap T)}{P(T)} = \frac{P(T|D)P(D)}{P(T)}$$

$$= \frac{\frac{900}{1000} \cdot \frac{1}{1000}}{P(T|D)P(D) + P(T|D^c)P(D^c)}$$

$$= \frac{\frac{900}{1000} \cdot \frac{1}{1000}}{\frac{900}{1000} \cdot \frac{1}{1000} + \frac{50}{1000}\frac{999}{1000}}$$

$$\frac{900}{900 + 49950} = \frac{900}{50850} \approx 0.0177$$