STA 256f19 Assignment Six[1]

Please read Sections 2.5 and 2.7. Also, look over your lecture notes. The following homework problems are not to be handed in. They are preparation for Term Test 2 and the final exam. Use the formula sheet.

1. Let X have an exponential density with parameter $\lambda > 0$. Find the cumulative distribution function $F_X(x)$.

2. Let X have a Normal(μ, σ^2) density. Express the cumulative distribution function $F_X(x)$ in terms of $\Phi(z)$, the cumulative distribution function of a Normal random variable with $\mu = 0$ and $\sigma^2 = 1$. The notation Φ is standard.

3. Do exercises 2.5.7, 2.5.9, 2.5.21, 2.5.23 in the text.

4. Let X be a random variable with cumulative distribution function $F_X(x)$

   (a) Prove $\lim\limits_{x \to \infty} F_X(x) = 1$.

   (b) Prove $\lim\limits_{x \to -\infty} F_X(x) = 0$.

5. Do Exercises 2.6.1, 2.6.3, 2.6.4, 2.6.5, 2.6.7 and 2.6.8 in the text. Please use the "distribution function technique" illustrated in lecture, and *not* theorems from Section 2.6. In particular, Theorems 2.6.1-2.6.4 are to be avoided and will not be on the formula sheet.

6. Let $p_{X,Y}(x, y) = c(x + y)$ for $x = 1, 2, 3$, $y = 1, 2$, and zero otherwise.

   (a) Find the constant c. [21]
   (b) What is $p_{X,Y}(1, 1)$? [2/21]
   (c) What is $p_{X,Y}(2.5, 1.75)$? [0]
   (d) What is $F_{X,Y}(2.5, 1.75)$? [5/21]
   (e) What is $F_{X,Y}(5, 1.5)$? [9/21]
   (f) What is $F_{X,Y}(0, 4)$? [0]
   (g) What is $F_{X,Y}(4, 4)$? [1]
   (h) What is $p_X(2)$? [7/21]
   (i) What is $p_Y(1)$? [9/21]
   (j) What is $F_X(2.5)$? [12/21]

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7. Do Exercise 2.7.1 in the text. Don't waste too much energy trying to understand the answer in the back of the book. Instead of giving the joint cumulative distribution function, answer the following questions instead. Notice that you are *not* being asked for a full statement of the cumulative distribution function.

   (a) Draw a set of x, y coordinates, plot all the points with non-zero probability, and write a probability beside each point.

   (b) What is $p_{X,Y}(x, y)$? Make sure your answer applies to all real x and y.

   (c) What is $F_{X,Y}(1, -1)$? I hope you agree it's $P(X = 0, Y = -2) = \frac{2}{3}$. Thus, the answer in the back of the book has to be wrong.

8. Do Exercise 2.7.3 in the text. It's easier if you put the joint probabilities in a two-way table.

9. Do Exercise 2.7.5 in the text. Hint: Think of sets contained in other sets.

10. Five cards are selected from an ordinary deck of 52 playing cards. Let X equal the number of spades and Y equal the number of hearts. Give the joint probability function of X and Y

    (a) If the sampling is *without* replacement. Be sure to specify the values of x and y for which $p_{X,Y}(x, y)$ is non-zero.

    (b) If the sampling is *with* replacement. Be sure to specify the values of x and y for which $p_{X,Y}(x, y)$ is non-zero

11. Please look at Example 2.7.3 on page 82 of the text.

    (a) Find the joint density $f_{X,Y}(x, y)$. Show your work. Make sure to specify where it is non-zero.

    (b) Check that your $f_{X,Y}(x, y)$ integrates to one.

    (c) Find the marginal density $f_X(x)$. If you do not specify where the density is non-zero, it's worth half marks at most.

    (d) Find the marginal density $f_Y(y)$. If you do not specify where the density is non-zero, it's worth half marks at most.

    (e) Find the marginal cumulative distribution function $F_X(x)$.

       i. By integrating $f_X(x)$.
       ii. By taking limits of $F_{X,Y}(x, y)$

       Make sure your answer applies to all real x.

(f) Find the marginal cumulative distribution function $F_Y(y)$.

    i. By integrating $f_Y(y)$.

    ii. By taking limits of $F_{X,Y}(x,y)$

Make sure your answer applies to all real y.

(g) Find $P(Y < X^2)$. Show your work. My answer is $\frac{1}{5}$.

12. Do Exercise 2.7.9 in the text. Hint: Sketch the support before beginning.][

13. Do Problem 2.7.12 in the text. Hint: Use Exercise 2.7.5.

14. Do Problem 2.7.16 in the text.