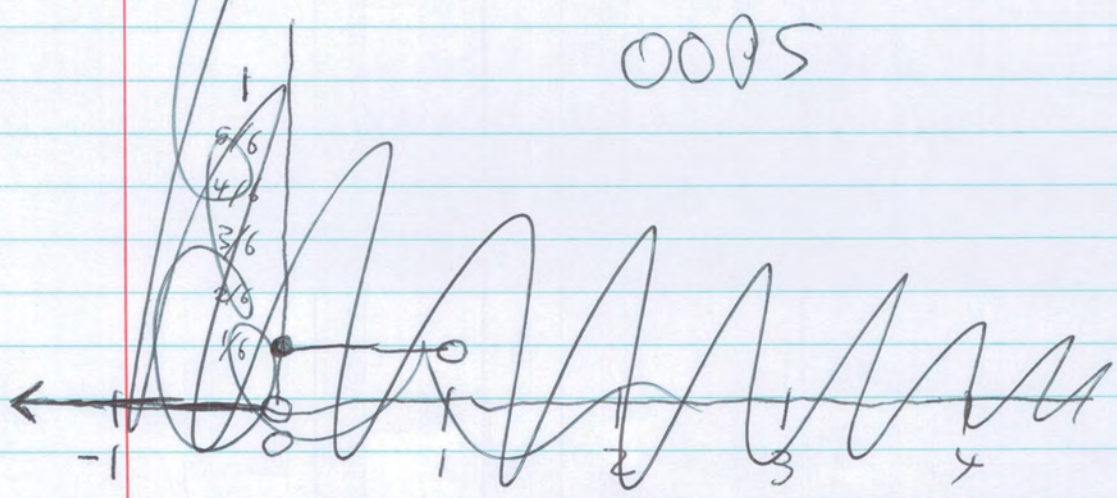
Assignment ~~3~~ 4

(2) Put the minimum in the cell, probability $\frac{1}{36}$ for each.

	1	2	3	4	5	6
1	1	1	1	1	1	1
2	1	2	2	2	2	2
3	1	2	3	3	3	3
4	1	2	3	4	4	4
5	1	2	3	4	5	5
6	1	2	3	4	5	6

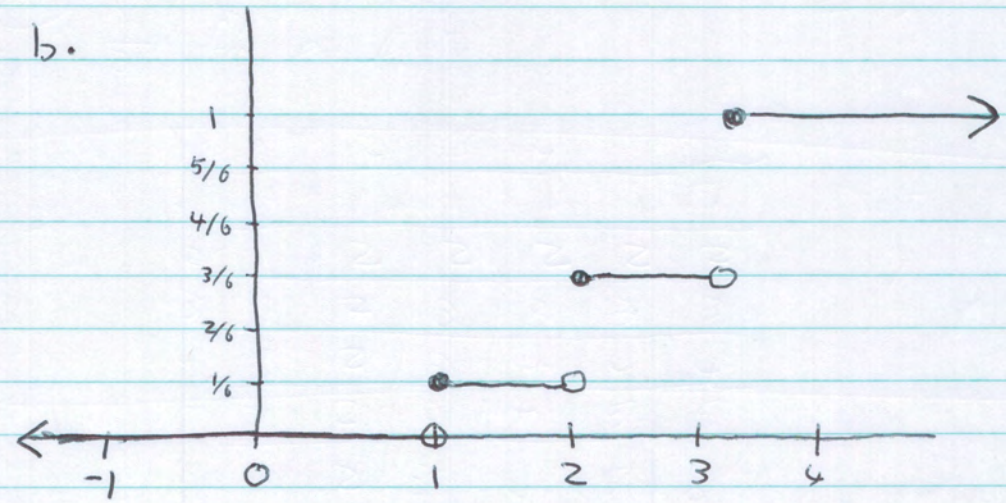
~~(3) a) $C_1(1+2+3) = 1$ = 2.5%~~

0005



(3) a. $c(1+2+3)=1 \Rightarrow c=\frac{1}{6}$

b.



c. $P(X \in B) =$

$$\frac{1}{6} I_B(1) + \frac{2}{6} I_B(2) + \frac{3}{6} I_B(3)$$

(5)

2.2.5 in the text

30 1s
20 2s
50 3s

c. only

In the table below, the probability in parentheses is $P(X=x, Y=y) = P(X=x)P(Y=y)$

		y = 1	2	3	$P(X=x)$
x	1	2 (0.09)	3 (0.06)	4 (0.15)	0.3
	2	3 (0.06)	4 (0.04)	5 (0.10)	0.2
	3	4 (0.15)	5 (0.10)	6 (0.25)	0.5
$P(Y=y)$		0.3	0.2	0.5	

The random variable $W = X + Y$ can assume the values 2, 3, 4, 5, 6. Using Property 4, add up the probabilities. Get the answer in the back of the book.

(6) $1 = \sum_x P(x) = c((-2)^2 + (-1)^2 + 0^2 + 1^2 + 2^2)$

$= c(4+1+1+4) = 10c$

$\Rightarrow c = \frac{1}{10}$

(7) $1 = \sum_{x=0}^{\infty} c \cdot \frac{2^x}{x!} = c \sum_{x=0}^{\infty} \frac{2^x}{x!} = c e^2$

Formula sheet

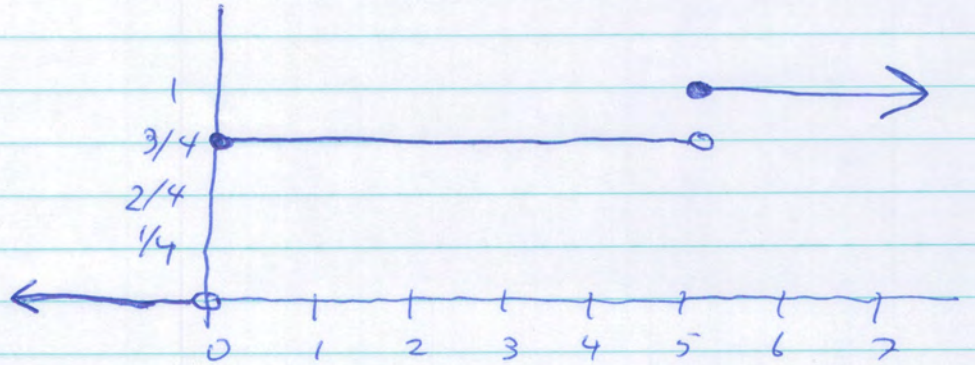
$\Rightarrow c = e^{-2}$

(9) (2.3.3 in the text). Cells show $z = xy$ $(P(X=x)P(Y=y))$

	$y=0$	$y=5$	
$x=0$	0 $(1/4)$	0 $(1/4)$	$\frac{1}{2}$
$x=1$	0 $(1/4)$	5 $(1/4)$	$\frac{1}{2}$
	$\frac{1}{2}$	$\frac{1}{2}$	

$$P_X(x) = \begin{cases} 3/4 & \text{for } x=0 \\ 1/4 & \text{for } x=5 \\ 0 & \text{otherwise} \end{cases}$$

$$F_X(x) = \begin{cases} 0 & \text{for } x<0 \\ 3/4 & \text{for } 0 \le x < 5 \\ 1 & \text{for } x \ge 5 \end{cases}$$



(10) (2.3.7 in text)

$$f(\theta) = \ln p(x) = \ln\left(\binom{12}{x} \theta^x (1-\theta)^{12-x} \right)$$

$$= \ln\left[\binom{12}{11} \theta^{11} (1-\theta) \right]$$

$$= \ln 12 + 11 \ln \theta + \ln(1-\theta)$$

$$\frac{df}{d\theta} = \frac{11}{\theta} - \frac{1}{1-\theta} \overset{\text{set}}{=} 0 \iff \frac{11}{\theta} = \frac{1}{1-\theta}$$

$$\iff \theta = 11(1-\theta) = 11 - 11\theta$$

$$\iff 12\theta = 11 \iff \theta = \frac{11}{12}$$

Is it really a ~~minimum~~ maximum?

$$\frac{d^2 f}{d\theta^2} = \frac{d}{d\theta}\left(11\theta^{-1} - (1-\theta)^{-1} \right)$$

$$= -11\theta^{-2} - -(1-\theta)^{-2} \cdot -1$$

$$= \frac{-11}{\theta^2} - \frac{1}{(1-\theta)^2} < 0 \quad \text{concave down}$$

$\cap$ MAX

Show Probabilities sum to one

(11) Bernoulli: $\sum_{x=0}^{1} \theta^x (1-\theta)^{1-x}$

$= 1 - \theta + \theta = 1$

(12) Binomial: $\sum_{x=0}^{n} \binom{n}{x} \theta^x (1-\theta)^{n-x}$

Using $a = \theta$ & $b = 1-\theta$ in the binomial formula

$= (\theta + 1 - \theta)^n = 1^n = 1$

(13) Geometric: $\sum_{x=0}^{\infty} (1-\theta)^x \theta = \theta \sum_{x=0}^{\infty} (1-\theta)^x$

Using geometric sum on formula sheet

$= \theta \cdot \frac{(1-\theta)^0}{1-(1-\theta)} = \frac{\theta}{\theta} = 1$

(14) Poisson: $\sum_{x=0}^{\infty} \frac{e^{-\lambda} \lambda^x}{x!} = e^{-\lambda} \sum_{x=0}^{\infty} \frac{\lambda^x}{x!}$

Formula sheet

$\overset{\downarrow}{=} e^{-\lambda} e^{\lambda} = 1$

(15) $X \sim G(\theta)$. Use $\sum_{k=j}^{\infty} a^k = \frac{a^j}{1-a}$

(a) $P(X > x) = \sum_{k=x}^{\infty} (1-\theta)^k \theta = \theta \sum_{k=x}^{\infty} (1-\theta)^k$

$$= \theta \cdot \frac{(1-\theta)^x}{1-(1-\theta)} = (1-\theta)^x$$

(b) $P(X > k+x \mid X > k)$

$$= \frac{P(X > x+k \cap X > k)}{P(X > k)} = \frac{P(X > k+x)}{P(X > k)}$$

$$\overset{\text{using Part (a)}}{=} \frac{(1-\theta)^{x+k}}{(1-\theta)^k} = (1-\theta)^x$$

(16) 2.3.15 in the text $P(\text{Basket}) = 0.35$

a) $X \sim \text{binomial}(n=10, \theta=0.35)$

$$P(X=3) = \binom{10}{3}\theta^3(1-\theta)^{10-3}$$

$$= 120 \cdot (0.35)^3 (0.65)^7$$ Book's answ.

$=$

b) $X \sim \text{Geometric}(\theta = 0.35)$ # failures before 1st success

P(10 shots before 1st basket)

$= P(X=9) = (0.65)^9(0.35)$

c) $X \sim \text{Negative binomial}(\theta=0.35, r=2)$

failures before rth success

P(10 shots before 2nd basket)

$= P(8 \text{ misses in 1st 9 shots, then hit})$

$= \binom{9}{8} 0.65^8\, 0.35 \cdot 0.35$, or

$$P(X=8) = \binom{x+r-1}{x}\theta^r(1-\theta)^x$$

$$= \binom{8+2-1}{8}(.35)^2(.65)^8$$

$$= \binom{9}{8}(0.65)^8(0.35)^2$$

as above

Book's answ.

(18) 2.3.19 in text

999w
1B

$n = 100$, with replacement

Poisson approximation to binomial

$$X \sim \text{Binomial}\left(n = 100,\ \theta = \frac{1}{1{,}000}\right)$$

Approximately Poisson $\left(\lambda = n\theta = \frac{1}{10}\right)$

$P(X=5)$ is approximately $\dfrac{e^{-\lambda}\lambda^{x}}{x!}$

$$= \frac{e^{-0.1}(0.1)^5}{5!} = 7.54 \times 10^{-8}$$

For comparison, the exact answer is around 6.85×10^{-8}

Using R's dbinom function