STA 256f19 Assignment Four[1]

Please read Sections 2.1-2.3 (pages 34-53) in the text, and look over your lecture notes. These homework problems are not to be handed in. They are preparation for Term Test 2 and the final exam.

1. Do exercises 2.1.1 and 2.1.9. For 2.1.9, see Example 2.1.6 on indicator functions.

2. Roll two fair dice, and let X be the minimum of the two numbers showing.

   (a) Give $p(x)$ and $F(x)$ for $x = 1, \ldots, 6$.

x	1	2	3	4	5	6
$p(x)$	11/36	9/36	7/36	5/36	3/36	1/36
$F(x)$	11/36	20/36	27/36	32/36	35/36	36/36

   (b) What is $F(1)$? (11/36)

   (c) What is $F(3)$? (27/36)

   (d) What is $F(3.5)$? (27/36)

   (e) What is $p(3.5)$? (0)

   (f) What is $F(0)$? (0)

   (g) What is $p(0)$? (0)

   (h) What is $F(-14)$? (0)

   (i) What is $F(14)$? (1)

   (j) What is $p(14)$? (0)

3. Let $p_X(x) = cx$ for $x = 1, 2, 3$ and zero otherwise.

   (a) What is the constant c? (1/6)

   (b) Graph the cumulative distribution function $F_X(x)$. Don't forget right continuity.

   (c) Write the full probability distribution of X using indicator functions as in Examples 2.2.2 and 2.2.3 in the text. For any set $B \in \mathbb{R}$, $P(X \in B) = \ldots$

4. Do exercise 2.2.7 in the text.

5. Do exercise 2.2.5 in the text. Assume the chips are mixed thoroughly, so that X and Y are independent.

6. Let the discrete random variable X have probability mass function $p(x) = c\,x^2$ for $x = -2, -1, 0, 1, 2$ and zero otherwise. What is the constant c? (1/10)

7. Let the discrete random variable X have probability mass function $p(x) = c\,\frac{2^x}{x!}$ for $x = 0, 1, \ldots$ and zero otherwise. What is the constant c? (e^{-2})

8. Do exercise 2.3.1 in the text. (See the Sample Questions).

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9. Do exercise 2.3.3 in the text. Also, what is the cumulative distribution function $F(x)$? Make sure your answer applies to all real x. Graph the cumulative distribution function $F(x)$. Don't forget right continuity.

10. Do exercise 2.3.7 in the text. Hint: Differentiate $\ln(p(x))$.

11. Show that the Bernoulli probabilities sum to one. Use the formula sheet.

12. Show that the Binomial probabilities sum to one. Use the formula sheet.

13. Show that the Geometric probabilities sum to one. Use the formula sheet.

14. Show that the Poisson probabilities sum to one. Use the formula sheet.

15. Let X be a geometric random variable with parameter θ.

    (a) Find $P(X \geq x)$ (Answer is $(1-\theta)^x$)

    (b) Find $P(X \geq x+k|X \geq k)$ (Answer is $(1-\theta)^x$)

    This is called the "memoryless" property of the geometric distribution. It makes sense in terms of coin tossing.

16. Do exercise 2.3.15 in the text.

17. Let X be a binomial (n, θ) random variable. Let $n \to \infty$ and $\theta \to 0$ in such a way that the value of $n\theta = \lambda$ remains fixed. Show that the probability mass function of X approaches the probability mass function of a Poisson as $n \to \infty$. This is called the Poisson approximation to the binomial. (See the Sample Questions.)

18. Do exercise 2.3.19 in the text.

The answer to Question 3c is $P(X \in B) = \frac{1}{6}I_B(1) + \frac{2}{6}I_B(2) + \frac{3}{6}I_B(3)$.
