Name ______________________________

Student Number ______________________________

STA 256 f2019 LEC0101 Test 3

Tutorial Section (Circle One)

TUT0101 Mon. 3-4 DH 2080 Ali	TUT0102 Mon. 4-5 DH 2080 Dashvin	TUT0103 Mon. 5-6 IB 360 Dashvin	TUT0104 Mon. 6-7 IB 240 Ali	TUT0105 Wed. 4-5 IB360 Marie
TUT0106 Wed. 5-6 IB 360 Marie	TUT0107 Fri. 9-10 IB 200 Crendall	TUT0108 Fri. 10-11 DH 2070 Ali	TUT0109 Fri. 10-11 DV 3093 Cendall	TUT0110 Fri. 11-12 DH 2070 Ali
TUT0111 Fri. 11-12 DV 3093 Crendall	TUT0112 Fri. 12-1 DV 2070 Crendall	TUT0113 Fri. 4-5 DV 3093 Karan	TUT0114 Fri. 5-6 IB 360 Karan	TUT0115 Fri. 6-7 IB 360 Karan
TUT0116 Wed. 11-12 DH 2070 Ana	TUT0117 Wed. 12-1 IB 260 Ana			

Question	Value	Score
1	20	
2	15	
3	25	
4	20	
5	20	
Total = 100 Points		

1. Let the random variables X and Y have joint density

$$f_{X,Y}(x,y) = \begin{cases} 24\,xy & \text{For } 0 \leq x \leq 1,\ 0 \leq y \leq 1 \text{ and } x+y \leq 1 \\ 0 & \text{Otherwise} \end{cases}$$

   (a) (3 points) Sketch the region of the x, y plane where the joint density is non-zero.

   (b) (8 points) Find the marginal density $f_X(x)$. Show your work. Be sure to specify where the density is non-zero.

Continued on page 3

(c) (1 point) You know the marginal density $f_Y(y)$ by symmetry. Just write it down. Be sure to specify where the density is non-zero.

(d) (2 points) Are X and Y independent? Answer Yes or No and briefly justify your answer.

(e) (6 points) Give the conditional density $f_{Y|X}(y|x)$. Be sure to specify where the density is non-zero.

Continued on page 4

2. (15 points) Let X and Y be independent, discrete random variables. Show that $E\{g(X)h(Y)\} = E\{g(X)\}\,E\{h(Y)\}$. Because X and Y are discrete, you will add rather than integrating to do this question. Be very clear about where you use independence. *Draw an arrow pointing to where you use independence, and write "This is where I use independence."*

Continued on page 5

3. Let X_1 and X_2 be independent normal random variables with $\mu = 0$ and $\sigma^2 = 1$. Let $Y_1 = X_1 - X_2$ and $Y_2 = X_1 + X_2$.

   (a) (10 points) Calculate the joint density of Y_1 and Y_2. Show your work, and **circle your final answer**. The next part of this question will go better if you simplify your answer.

Continued on page 6

Continue Question 3 if necessary.

Continued on page 7

(b) (10 points) Find the marginal density of $Y_1 = X_1 - X_2$.

(c) (**5 points**) The distribution of Y_1 is one of the common distributions on the formula sheet. Identify it by name and give the value(s) of the parameter(s).

Continued on page 8

4. Let X have a binomial distribution with parameters n and θ.

   (a) (12 points) Derive the moment-generating function of X. Show your work. **Circle your final answer**. You can check your answer on the formula sheet, but if you force your answer to come out "right" by making a convenient mistake, you will get a zero for this part.

   (b) (8 points) Use the moment-generating function to find $E(X)$. Show your work. **Circle your answer**. If your answer to part (a) does not agree with the formula sheet, use the formula sheet.

Continued on page 9

5. (20 points) Let X_1, X_2 and X_3 be independent random variables, where

$$X_1 \sim \text{Gamma}\ (\alpha = 1, \lambda = 1) \quad X_2 \sim \text{Gamma}\ (\alpha = 2, \lambda = 1) \quad X_3 \sim \text{Gamma}\ (\alpha = 3, \lambda = 1)$$

Find the distribution of $Y = X_1 + X_2 + X_3$. Show your work. It is one of the common distributions on the formula sheet. *Name the distribution and give the values of the parameters.*

Total Marks = 100 points