Revised STA 256 Formulas

$$\sum_{k=j}^{\infty} a^k = \frac{a^j}{1-a} \qquad \sum_{k=0}^{\infty} \frac{x^k}{k!} = e^x \qquad (a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$$

$\lim_{x \to c} \frac{g(x)}{h(x)} = \lim_{x \to c} \frac{g'(x)}{h'(x)}$ if $\frac{0}{0}$ or $\frac{\infty}{\infty}$ etc.

$$\lim_{n\to\infty} \left(1 + \frac{x}{n}\right)^n = e^x \qquad \Gamma(\alpha) = \int_0^\infty e^{-t} t^{\alpha-1}\, dt \qquad \Gamma(\alpha+1) = \alpha\,\Gamma(\alpha)$$

Distributive Laws of Sets: $A \cap \left(\cup_{j=1}^\infty B_j\right) = \cup_{j=1}^\infty (A \cap B_j)$ $\quad A \cup \left(\cap_{j=1}^\infty B_j\right) = \cap_{j=1}^\infty (A \cup B_j)$

De Morgan Laws: $(\cap_{j=1}^\infty A_j)^c = \cup_{j=1}^\infty A_j^c$ $\quad (\cup_{j=1}^\infty A_j)^c = \cap_{j=1}^\infty A_j^c$

Axioms: (1) $P(\Omega) = 1$ (2) $P(A) \geq 0$ for any $A \subset \Omega$
(3) If $A_1, A_2 \ldots$ are disjoint, $P\left(\cup_{k=1}^\infty A_k\right) = \sum_{k=1}^\infty P(A_k)$

Properties: **A**. $P(A^c) = 1 - P(A)$ **C**. If $A \subseteq B$ then $P(A) \leq P(B)$
D. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$${}_nP_r = \frac{n!}{(n-r)!} \qquad \binom{n}{r} = \frac{n!}{r!\,(n-r)!} \qquad \binom{n}{n_1 \cdots n_k} = \frac{n!}{n_1! \cdots n_k!}$$

$$P(A|B) \overset{def}{=} \frac{P(A\cap B)}{P(B)} \qquad P(A \cap B) = P(A|B)P(B) \qquad P(A) = \sum_{k=1}^\infty P(A|B_k)P(B_k)$$

$$P(B_j|A) = \frac{P(A|B_j)P(B_j)}{\sum_{k=1}^\infty P(A|B_k)P(B_k)} \qquad P(B|A) = \frac{P(A|B)P(B)}{P(A|B)P(B) + P(A|B^c)P(B^c)}.$$

A and B independent means $P(A \cap B) = P(A)P(B)$.

Distribution	**Density or probability mass function**	**MGF** M(t)
Bernoulli (p)	$p(x) = p^x(1-p)^{1-x}$ for $x = 0, 1$	$pe^t + 1 - p$
Binomial (n, p)	$p(k) = \binom{n}{k} p^k (1-p)^{n-k}$ for $k = 0, 1, \ldots, n$	$(pe^t + 1 - p)^n$
Geometric (p)	$p(k) = (1-p)^{k-1} p$ for $k = 1, 2, \ldots$	$p(e^{-t} + p - 1)^{-1}$
Negative Binomial (r, p)	$\binom{k-1}{r-1} p^r (1-p)^{k-r}$ for $k = r, r+1, \ldots$	
Hypergeometric (n, r, m)	$p(k) = \frac{\binom{r}{k}\binom{n-r}{m-k}}{\binom{n}{m}}$ for $k = 0, \ldots r$	
Poisson (λ)	$p(k) = \frac{e^{-\lambda}\lambda^k}{k!}$ for $k = 0, 1, \ldots$	$e^{\lambda(e^t - 1)}$
Multinomial $(n, p_1, \ldots, p_r)$	$p(n_1, \ldots, n_r) = \binom{n}{n_1 \cdots n_r} p_1^{n_1} \cdots p_r^{n_r}$	
Uniform (a, b)	$f(x) = \frac{1}{b-a}$ for $a \leq x \leq b$	$\frac{e^{bt} - e^{at}}{t(b-a)}$ for $t \neq 0$
Exponential (λ)	$f(x) = \lambda e^{-\lambda x}$ for $x \geq 0$	$(1 - \frac{t}{\lambda})^{-1}$
Gamma (α, λ)	$f(x) = \frac{\lambda^\alpha}{\Gamma(\alpha)} e^{-\lambda x} x^{\alpha-1}$ for $x \geq 0$	$(1 - \frac{t}{\lambda})^{-\alpha}$
Normal (μ, σ)	$\frac{1}{\sigma\sqrt{2\pi}} \exp - \left\{\frac{(x-\mu)^2}{2\sigma^2}\right\}$ $\quad \frac{X-\mu}{\sigma} \sim N(0,1)$	$e^{\mu t + \frac{1}{2}\sigma^2 t^2}$
Chi-squared (ν)	$\frac{1}{2^{\frac{\nu}{2}}\Gamma(\frac{\nu}{2})} e^{-\frac{x}{2}} x^{\frac{\nu}{2}-1}$ for $x \geq 0$	$(1-2t)^{-\frac{\nu}{2}}$
Beta	$f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}$ for $0 \leq x \leq 1$	

$F_x(x) \stackrel{def}{=} P(X \leq x)$ $\quad$ $F_{xy}(x,y) \stackrel{def}{=} P(X \leq x, Y \leq y)$

$F_{xy}(x,y) = \int_{-\infty}^{x} \int_{-\infty}^{y} f_{xy}(s,t)\, dt\, ds$ $\quad$ $f_{xy}(x,y) = \frac{\partial^2}{\partial x \partial y} F_{xy}(x,y)$

$F_x(x) = \lim_{y \to \infty} F_{xy}(x,y)$ $\quad$ $f_x(x) = \int_{-\infty}^{\infty} f_{xy}(x,y)\, dy$

$p_x(x) \stackrel{def}{=} P(X = x))$ $\quad$ $p_x(x) = \sum_y p_{xy}(x,y)$

Independence: $F_{xy}(x,y) = F_x(x)F_y(y) \quad \Leftrightarrow \quad p_{xy}(x,y) = p_x(x)\, p_y(y)$ or $f_{xy}(x,y) = f_x(x)\, f_y(y)$

$p_{y|x}(y|x) \stackrel{def}{=} \frac{p_{x,y}(x,y)}{p_x(x)}$ $\quad$ $f_{x|y}(x|y) \stackrel{def}{=} \frac{f_{x,y}(x,y)}{f_y(y)}$

Convolution formulas: If X and Y are independent random variables, and $Z = X + Y$

$p_z(z) = \sum_x p_x(x) p_y(z - x)$ $\quad$ $f_z(z) = \int_{-\infty}^{\infty} f_x(x) f_y(z - x)\, dx$

Jacobian formula: $Y_1 = g_1(X_1, X_2)$ and $Y_2 = g_2(X_1, X_2)$

$$f_{y_1 y_2}(y_1, y_2) = f_{x_1 x_2}(\, x_1(y_1,y_2), x_2(y_1,y_2)\,) \cdot abs \begin{vmatrix} \frac{\partial x_1}{\partial y_1} & \frac{\partial x_1}{\partial y_2} \\ \frac{\partial x_2}{\partial y_1} & \frac{\partial x_2}{\partial y_2} \end{vmatrix} = f_{x_1 x_2}(\, x_1(y_1,y_2), x_2(y_1,y_2)\,) \left| \frac{\partial x_1}{\partial y_1}\frac{\partial x_2}{\partial y_2} - \frac{\partial x_1}{\partial y_2}\frac{\partial x_2}{\partial y_1} \right|$$

$dx\, dy = r\, dr\, d\theta$

$E(X) \stackrel{def}{=} \sum_x x\, p_x(x)$ or $\int_{-\infty}^{\infty} x\, f_x(x)\, dx$ $\quad$ $E(g(X)) = \sum_x g(x)\, p_x(x)$ or $\int_{-\infty}^{\infty} g(x)\, f_x(x)\, dx$

$E\left(\sum_{i=1}^n a_i X_i\right) = \sum_{i=1}^n a_i E(X_i)$ $\quad$ $E(X) = E(E[X|Y])$

$Var(X) \stackrel{def}{=} E\left((X - \mu)^2\right)$ $\quad$ $Var(X) = E(X^2) - [E(X)]^2$

$Var(a + bX) = b^2 Var(X)$ $\quad$ $Var(aX + bY) = a^2 Var(X) + b^2 Var(Y) + 2ab Cov(X,Y)$

$Cov(X,Y) \stackrel{def}{=} E[(X - \mu_x)(Y - \mu_y)]$ $\quad$ $Cov(X,Y) = E(XY) - E(X)E(Y)$

$Cov(a + bX, c + dY) = bd\, Cov(X,Y)$ $\quad$ $Cov(X, aY + bZ) = a\, Cov(X,Y) + b\, Cov(X,Z)$

$Var\left(\sum_{i=1}^n a_i X_i\right) = \sum_{i=1}^n a_i^2 Var(X_i) \, + \, \sum\sum_{i \neq j} a_i b_j Cov(X_i, X_j)$

Markov's inequality $\quad$ Chebyshev's inequality

If $P(Y \geq 0) = 1$, then $P(Y \geq t) \leq E(Y)/t$ $\quad$ $P(|X - \mu| \geq k\sigma) \leq \frac{1}{k^2}$

$M(t) \stackrel{def}{=} E(e^{Xt})$ $\quad$ $M^{(k)}(0) = E(X^k)$

$M_{aX}(t) = M_X(at)$ $\quad$ $M_{\sum X_i}(t) = \prod_{i=1}^n M_{X_i}(t)$ if the X_i are independent

Law of Large Numbers: For all $\epsilon > 0$, $\lim_{n \to \infty} P\{|\overline{X}_n - \mu| \geq \epsilon\} = 0$.

Central Limit Theorem: $Z_n = \frac{\sqrt{n}(\overline{X}_n - \mu)}{\sigma}$ converges in distribution to a standard normal.

Glossary

CDF: Cumulative Distribution Function $F(x) = P(X \leq x)$

Central Limit Theorem (CLT): Sample mean is approximately normal for large samples; see formula sheet for details.

Conditional density: The density of continuous X given that continuous Y equals y. $f_{x|y}(x|y)$. Probability is area under this curve. To get probabilities and expected values, integrate.

Conditional probability mass function: The PMF of discrete X given that discrete Y equals y. $p_{x|y}(x|y) = P(X = x|Y = y)$. To get probabilities and expected values, add.

Continuous random variable: X assumes an uncountably infinte number of value. Probability is area under the curve $f(x)$. Integrate to find probabilities and expected values.

Convergence in Distribution: A sequence of cumulative distribution functions converges to a target cumulative distribution function at all continuity points of the target.

Convolution: X and Y are independent, $Z = X + Y$. The convolution formulas are $p_z(z) = \sum_x p_x(x)p_y(z - x)$ and $f_z(z) = \int_{-\infty}^{\infty} f_x(x)f_y(z - x)\,dx$,

Density: Probability density function $f(x)$. Probability of a continuous random variable is area under this curve. To get probabilities and expected values, integrate.

Discrete random variable: X assumes a finite or countably infinite number of values. Add to find probabilities and expected values.

Disjoint: Mutually exclusive, non-overlapping, $A \cap B = \emptyset$.

Frequency function: Same as a probability mass function $p(x) = P(X = x)$. Applies to discrete random variables. To get probabilities and expected values, add.

Joint density: Applies to continuous random variables. $f_{xy}(x, y)$. Probability is volume under this surface. To get probabilities or expected values, integrate.

Joint probability mass function: Applies to discrete random variables. $p_{xy}(x, y) = P(X = x, Y = y)$. To get probabilities or expected values, add.

Joint frequency function: Same as joint probability mass function.

Marginal density: The density of one of the continuous random variables along the edge (margin), integrating over the other one. $f_x(x) = \int_{-\infty}^{\infty} f_{xy}(x, y)\,dy$.

Marginal probability mass function: The PMF of one of the discrete random variables along the edge (margin), adding over the other one. $p_x(x) = \sum_y p_{xy}(x, y)$

Moment-generating function: $M(t) = E(e^{Xt})$. Corresponds uniquely to the probability distribution function of X. To get $E(X^k)$, differentiate k times and set $t = 0$.

MGF: Moment-Generating Function. See above.

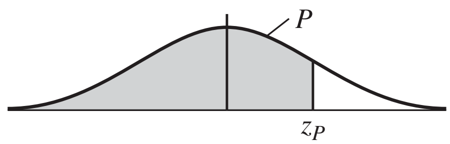
PDF: Probability density function $f(x)$. Probability is area under this curve. To get probabilities and expected values, integrate.

PMF: Probability mass function $p(x) = P(X = x)$, same as a frequency function. Applies to discrete random variables. To get probabilities and expected values, add.

CBC: Canadian Broadcasting Corporation

The table on the reverse side is from *Mathematical statistics and data analysis* by Rice.

TABLE 2 Cumulative Normal Distribution—Values of P Corresponding to z_p for the Normal Curve



z is the standard normal variable. The value of P for $-z_p$ equals 1 minus the value of P for $+z_p$; for example, the P for –1.62 equals 1 – .9474 = .0526.

z_p	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998