STA2112f99 Assignment for Quiz 5

1. Prove Theorem 2.2.1 for a mixed random variable.

2. Prove $Var(Y) = E(Y^2) - [E(Y)]^2$ for a mixed random variable.

3. Let Y be a mixed random variable with discrete part X_1 and continuous part X_2. Is this correct? $Var(Y) = \alpha Var(X_1) + (1-\alpha)Var(X_2)$. If it is correct, prove it. If not, give a convenient formula in terms of the means and variances of the component variables. Simplify.

4. Prove that if $P(0 \leq Y \leq 1) = 1$, then $Var(Y) \leq \frac{1}{4}$.

5. Let $f_X(x) = \theta(1-\theta)^{x-1}\, I\{x = 1, 2, \ldots\}$. For what values of t does $E(e^{Xt})$ exist? Does $M_X(t)$ exist?

6. Let $f_X(x) = \frac{1}{x^2}\, I\{x > 1\}$. For what values of t does $E(e^{Xt})$ exist? Does $M_X(t)$ exist?

7. Give an example of a non-negative random variable for which $E(X) < P(X \geq 1)$, or else prove that no such random variable is possible.

8. Prove that if $E(Y^2)$ exists, then $E(Y)$ must exist.

9. Let $f_X(x) = \alpha x^{\alpha-1} I\{0 < x < 1\}$, where $\alpha > 0$. Differentiate both sides of $\int f_X(x)\, dx = 1$ to find a formula for $E[\log(X)]$. How do you justify the exchange of differentiation and integration?

10. Let X have a geometric distribution (see above). For what values of θ does the infinite sum for $E(\frac{\cos(X)}{X^2})$ converge uniformly?