Exercise on Conditional Expectation

For probability model $(\Omega, \mathcal{A}, P)$, where $Y : (\Omega, \mathcal{A}) \rightarrow (\mathbb{R}^1, \mathcal{B}^1)$ is a random variable with $E(|Y|) < \infty$ and $\mathcal{D} \subset \mathcal{C} \subset \mathcal{A}$ are σ-algebras. Then establish the following and recall that equality here means equal with probability 1.

(i) $E(Y \mid \mathcal{A}) = Y$.

(ii) $E(Y \mid \{\phi, \Omega\}) = E(Y)$

(iii) $E(E(Y \mid \mathcal{D}) \mid \mathcal{C}) = E(Y \mid \mathcal{D})$.

(iv) $E(E(Y \mid \mathcal{C}) \mid \mathcal{D}) = E(Y \mid \mathcal{D})$.