Appendix E

Answers to Odd-Numbered Exercises

Answers are provided here to odd-numbered exercises that require a computation. No details of the computations are given. If the Exercise required that something be demonstrated, then a significant hint is provided.

1.2.1 (a) $P(\{1,2\}) = 5/6$ (b) $P(\{1,2,3\}) = 1$ (c) $P(\{1\}) = P(\{2,3\}) = 1/2$

1.2.3 $P(\{2\}) = 1/6$

1.2.5 $P(\{s\}) = 0$ for any $s \in [0,1]$

1.2.7 This is the subset $(A \cap B^c) \cup (A^c \cap B)$.

1.2.9 $P(\{1\}) = 1/12$, $P(\{2\}) = 1/12$, $P(\{3\}) = 1/6$, $P(\{4\}) = 2/3$

1.2.11 $P(\{2\}) = 5/24$, $P(\{1\}) = 3/8$, $P(\{3\}) = 5/12$

1.3.1 (a) $P(\{2,3,4,\ldots,100\}) = 0.9$ (b) 0.1

1.3.3 $P(\text{late or early or both}) = 25\%$

1.3.5 (a) $1/32 = 0.03125$. (b) 0.96875

1.3.7 10%

1.4.1 (a) $(1/6)^8 = 1/1679616$ (b) $(1/6)^7 = 1/279936$ (c) $8\,(1/6)^8 = 1/209952$

1.4.3 $1 - 5051/2^{100}$

1.4.5 (a) $\binom{4}{1}\binom{13}{13}\binom{39}{13\ 13\ 13} / \binom{52}{13\ 13\ 13\ 13}$ (b) $\binom{4}{1}\binom{4}{4}\binom{48}{9}\binom{39}{13\ 13\ 13} \,/\, \binom{52}{13\ 13\ 13\ 13}$

1.4.7 $\binom{48}{10} \,/\, \binom{52}{10} = 246/595 = 0.4134$

1.4.9 $(5/6)^2(1/6) = 25/216$

1.4.11 $\left(\binom{5}{3} / \binom{12}{3}\right)\left(\binom{6}{3} / \binom{18}{3}\right) + \left(\binom{7}{3} / \binom{12}{3}\right)\left(\binom{12}{3} / \binom{18}{3}\right)$

1.4.13 $\binom{2}{1}\frac{1}{2^2} \cdot \binom{3}{2}\frac{1}{2^3} \cdot \binom{4}{2}\frac{1}{2^4} + \binom{2}{1}\frac{1}{2^2} \cdot \binom{3}{0}\frac{1}{2^3} \cdot \binom{4}{3}\frac{1}{2^4} = \frac{11}{128} = 0.0859$

1.5.1 (a) $3/4$ (b) $16/21$

1.5.3 (a) $1/8$ (b) $(1/8)/(1/2) = 1/4$ (c) $0/(1/2) = 0$

1.5.5 1

1.5.7 0.074

1.5.9 (a) No (b) Yes (c) Yes (d) Yes (e) No

1.5.11 (a) 0.1667 (b) 0.3125

1.6.1 $1/3$

1.6.3 $\{A_n\} \nearrow A = \{1, 2, 3, \ldots\} = S$

1.6.5 1

1.6.7 Suppose there is no n such that $P([0, n]) > 0.9$ and then note this implies $1 = P([0, \infty)) = \lim_{n\to\infty} P([0, n]) \le 0.9$.

1.6.9 Yes

2.1.1 (a) 1 (b) Does not exist (c) Does not exist (d) 1

2.1.3 (a) $X(s) = s$ and $Y(s) = s^2$ for all $s \in S$. (b) For this example, $Z(1) = 2$, $Z(2) = 18$, $Z(3) = 84$, $Z(4) = 260$, $Z(5) = 630$.

2.1.5 Yes, for $A \cap B$.

2.1.7 (a) $W(1) = 1$ (b) $W(2) = 0$ (c) $W(3) = 0$ (d) $W \ge Z$ is not true.

2.1.9 (a) $Y(1) = 1$ (b) $Y(2) = 4$ (c) $Y(4) = 0$

2.2.1 $P(X = 0) = P(X = 2) = 1/4$, $P(X = 1) = 1/2$, $P(X = x) = 0$ for $x \ne 0, 1, 2$

2.2.3 (a) $P(Y = y) = 0$ for $y \ne 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12$, $P(Y = 2) = 1/36$, $P(Y = 3) = 2/36$, $P(Y = 4) = 3/36$, $P(Y = 5) = 4/36$, $P(Y = 6) = 5/36$, $P(Y = 7) = 6/36$, $P(Y = 8) = 5/36$, $P(Y = 9) = 4/36$, $P(Y = 10) = 3/36$, $P(Y = 11) = 2/36$, $P(Y = 12) = 1/36$ (b) $P(Y \in B) = (1/36)I_B(2) + (2/36)I_B(3)+(3/36)I_B(4)+(4/36)I_B(5)+(5/36)I_B(6)+(6/36)I_B(7)+(5/36)I_B(8)+ (4/36)I_B(9) + (3/36)I_B(10) + (2/36)I_B(11) + (1/36)I_B(12)$

2.2.5 (a) $P(X = 1) = 0.3$, $P(X = 2) = 0.2$, $P(X = 3) = 0.5$, and $P(X = x) = 0$ for all $x \notin \{1, 2, 3\}$ (b) $P(Y = 1) = 0.3$, $P(Y = 2) = 0.2$, $P(Y = 3) = 0.5$, and $P(Y = y) = 0$ for all $y \notin \{1, 2, 3\}$ (c) $P(W = 2) = 0.09$, $P(W = 3) = 0.12$, $P(W = 4) = 0.34$, $P(W = 5) = 0.2$, $P(W = 6) = 0.25$, and $P(W = w) = 0$ for all other choices of w.

2.2.7 $P(X = 25) = 0.45$, $P(X = 30) = 0.55$, and $P(X = x) = 0$ otherwise

2.3.1 $p_Y(2) = 1/36$, $p_Y(3) = 2/36$, $p_Y(4) = 3/36$, $p_Y(5) = 4/36$, $p_Y(6) = 5/36$, $p_Y(7) = 6/36$, $p_Y(8) = 5/36$, $p_Y(9) = 4/36$, $p_Y(10) = 3/36$, $p_Y(11) = 2/36$, $p_Y(12) = 1/36$, and $p_Y(y) = 0$ otherwise

2.3.3 $p_Z(1) = p_Z(5) = 1/4$, $p_Z(0) = 1/2$, and $p_Z(z) = 0$ otherwise

2.3.5 $p_W(1) = 1/36$, $p_W(2) = 2/36$, $p_W(3) = 2/36$, $p_W(4) = 3/36$, $p_W(5) = 2/36$, $p_W(6) = 4/36$, $p_W(8) = 2/36$, $p_W(9) = 1/36$, $p_W(10) = 2/36$, $p_W(12) = 4/36$, $p_W(15) = 2/36$, $p_W(16) = 1/36$, $p_W(18) = 2/36$, $p_W(20) = 2/36$, $p_W(24) = 2/36$, $p_W(25) = 1/36$, $p_W(30) = 2/36$, and $p_W(36) = 1/36$, with $p_W(w) = 0$ otherwise

2.3.7 $\theta = 11/12$

2.3.9 $53/512$

2.3.11 θ^{10}

2.3.15 (a) $\binom{10}{3}(0.35)^3(0.65)^7$ (b) $(0.35)(0.65)^9$ (c) $\binom{9}{1}(0.35)^2(0.65)^8$

2.3.17 (a) Hypergeometric(9, 4, 2) (b) Hypergeometric(9, 5, 2)

2.3.19 $P(X = 5) \approx ((100/1000)^5/5!) \exp\{-100/1000\}$

2.4.1 (a) 0 (b) 0 (c) 0 (d) 2/3 (e) 2/3 (f) 1 (g) = 1

2.4.3 (a) e^{-20} (b) 1 (c) e^{-12} (d) e^{-25}

2.4.5 No

2.4.7 $c = 3/M^3$

2.4.9 $\int_1^2 f(x)\,dx > \int_1^2 g(x)\,dx$

2.4.11 Yes

2.4.13 $P(Y < 3) = \int_{-\infty}^3 (2\pi)^{-1/2} \exp(-(y-1)^2/2)\,dy = \int_{-\infty}^2 (2\pi)^{-1/2} \exp(-u^2/2)\,du = P(X < 2)$

2.5.1 Properties (a) and (b) follow by inspection. Properties (c) and (d) follow since $F_X(x) = 0$ for $x < 1$, and $F_X(x) = 1$ for $x > 6$.

2.5.3 (a) No (b) Yes (c) Yes (d) No (e) Yes (f) Yes (g) No

2.5.5 Hence: (a) 0.933 (b) 0.00135 (c) 1.90×10^{-8}

2.5.7 (a) 1/9 (b) 3/16 (c) 12/25 (d) 0 (e) 1 (f) 0 (g) 1 (h) 0

2.5.9 (b) No

2.5.11 (b) Yes

2.5.13 (b) The function F is nondecreasing, $\lim_{x\to-\infty} F(x) = 0$ and $\lim_{x\to\infty} F(x) = 1$. (c) $P(X > 4/5) = 0$, $P(-1 < X < 1/2)3/4$, $P(X = 2/5)$, $P(X = 4/5) = 1/4$

2.5.15 (a) $P(Z > 4/5) = 2e^{-4/5}/3$ (b) $P(-1 < Z < 1/2) = 11/12 - 2e^{-1/2}/3$ (c) $P(Z = 2/5) = 5/36$ (d) $P(Z = 4/5) = 1/12$ (e) $P(Z = 0) = 1/9$ (f) $P(Z = 1/2) = 11/12 - 2e^{-1/2}/3$

2.6.1 $f_Y(y)$ equals $1/(R-L)c$ for $L \le (y-d)/c \le R$ and otherwise equals 0

2.6.3 $f_Y(y) = e^{-[y-d-c\mu]^2/2c^2\sigma^2}/c\sigma\sqrt{2\pi}$

2.6.5 $f_Y(y)$ equals $(\lambda/3)y^{-2/3}e^{-\lambda y^{1/3}}$ for $y > 0$ and otherwise equals 0.

2.6.7 $f_Y(y) = 1/6y^{1/2}$ for $0 < y < 9$

2.6.9 (a) $f_Y(y) = y/8$ (b) $f_Z(z) = z^3/4$

2.6.11 $f_Y(y) = y^{-1/2}\sin(y^{1/2})/4$ for $y > 1$ and 0 otherwise

2.6.13 $f_Y(y) = (2\pi)^{-1/2}(3|y|^{2/3})^{-1}\exp(-|y|^{2/3}/2)$

2.7.1

$$F_{X,Y}(x,y) = \begin{cases} 0 & \min[x,\ (y+2)/4] < 0 \\ 1/3 & 0 \le \min[x,\ (y+2)/4] < 1 \\ 1 & \min[x,\ (y+2)/4] \ge 1 \end{cases}$$

2.7.3 (a) $p_X(2) = p_X(3) = p_X(-3) = p_X(-2) = p_X(17) = 1/5$, with $p_X(x) = 0$ otherwise (b) $p_Y(3) = p_Y(2) = p_Y(-2) = p_Y(-3) = p_Y(19) = 1/5$, with $p_Y(y) = 0$ otherwise (c) $P(Y > X) = 3/5$ (d) $P(Y = X) = 0$ (e) $P(XY < 0) = 0$

2.7.5 $\{X \le x,\ Y \le y\} \subseteq \{X \le x\}$ and $\{X \le x,\ Y \le y\} \subseteq \{Y \le y\}$

2.7.7 (a) $f_X(x) = c(1-\cos(2x))/x$ for $0 < x < 1$ and 0 otherwise (b) $f_Y(y) = c(1-\cos(y))/y$ for $0 < y < 2$ and 0 otherwise

2.7.9 (a) $f_X(x) = (4 + 3x^2 - 2x^3)/8$ for $x \in (0, 2)$ and 0 otherwise (b) $f_Y(y) = (y^3 + 3y^2)/12$ for $y \in (0, 2)$ and 0 otherwise (c) $P(Y < 1) = 5/48$

2.8.1 (a) $p_X(-2) = 1/4$, $p_X(9) = 1/4$, $p_X(13) = 1/2$, otherwise, $p_X(x) = 0$ (b) $p_Y(3) = 2/3$, $p_Y(5) = 1/3$; otherwise, $p_Y(y) = 0$ (c) Yes

2.8.3 (a) $f_X(x) = (18x/49) + (40/49)$ for $0 \le x \le 1$ and $f_X(x) = 0$ otherwise (b) $f_Y(y) = (48y^2 + 6y + 30)/49$ for $0 \le y \le 1$ and $f_Y(y) = 0$ otherwise (c) No

2.8.5 (a) $P(Y = 4 \mid X = 9) = 1/6$ (b) $P(Y = -2 \mid X = 9) = 1/2$ (c) $P(Y = 0 \mid X = -4) = 0$ (d) $P(Y = -2 \mid X = 5) = 1$ (e) $P(X = 5 \mid Y = -2) = 1/3$

2.8.7 (a) $f_X(x) = x^2 + 2/3$, $f_Y(y) = 4y^5 + 2y/3$ for $0 \le x \le 1$ and $0 \le y \le 1$, $f_{Y|X}(y \mid x) = (2x^2y + 4y^5) / (x^2 + 2/3)$ (otherwise, $f_{Y|X}(y \mid x) = 0$), thus, X and Y are not independent. (b) $f_X(x) = C(x^5/6 + x/2)$, $f_Y(y) = C(y^5/6 + y/2)$ for $0 \le x \le 1$ and $0 \le y \le 1$, $f_{Y|X}(y \mid x) = (xy + x^5y^5) / (x^5/6 + x/2)$ (otherwise, $f_{Y|X}(y \mid x) = 0$) X and Y are not independent. (c) $f_X(x) = C(50{,}000x^5/3 + 50x)$, $f_Y(y) = C(2048y^5/3 + 8y)$ for $0 \le x \le 4$ and $0 \le y \le 10$, $f_{Y|X}(y \mid x) = (xy + x^5y^5) / (50{,}000x^5/3 + 50x)$ (otherwise, $f_{Y|X}(y \mid x) = 0$), thus, X and Y are not independent. (d) $f_X(x) = C(50{,}000x^5/3)$ and $f_Y(y) = C(2048y^5/3)$ for $0 \le x \le 4$ and $0 \le y \le 10$, $f_{Y|X}(y \mid x) = 3y^5 / 50000$ (otherwise, $f_{Y|X}(y \mid x) = 0$), X and Y are independent.

2.8.9 $P(X = 1,\ Y = 1) = P(X = 1,\ Y = 2) = P(X = 2,\ Y = 1) = P(X = 3,\ Y = 3) = 1/4$

2.8.11 If $X = C$ is constant, then $P(X \in B_1) = I_{B_1}(C)$ and $P(X \in B_1,\ Y \in B_2) = I_{B_1}(C)\, P(Y \in B_2)$.

2.8.13 (a)

$p_{Y\mid X}(y\mid x)$	$y = 1$	$y = 2$	$y = 4$	$y = 7$	Others
$x = 3$	$1/4$	$1/4$	$1/4$	$1/4$	0
$x = 5$	$1/4$	$1/4$	$1/4$	$1/4$	0

(b)

$p_{X\mid Y}(x\mid y)$	$x = 3$	$x = 5$	Others
$y = 1$	$1/2$	$1/2$	0
$y = 2$	$1/2$	$1/2$	0
$y = 4$	$1/2$	$1/2$	0
$y = 7$	$1/2$	$1/2$	0

(c) X and Y are independent.

2.8.15 $f_{Y|X}(y|x) = 2(x^2 + y)/(4 + 3x^2 - 2x^3)$ for $x < y < 2$, and 0 otherwise (b) $f_{X|Y}(x|y) = 3(x^2 + y)/(y^3 + 3y^2)$ for $0 < x < y$ and 0 otherwise (c) Not independent

2.9.1 $\frac{\partial h_1}{\partial u_1} = -\cos(2\pi u_2) / u_1\sqrt{2\log(1/u_1)}$, $\frac{\partial h_1}{\partial u_2} = -2\sqrt{2}\pi \sin(2\pi u_2)\sqrt{2\log(1/u_1)}$, $\frac{\partial h_2}{\partial u_1} = -\sin(2\pi u_2) / u_1\sqrt{2\log(1/u_1)}$, $\frac{\partial h_2}{\partial u_2} = -2\sqrt{2}\pi \cos(2\pi u_2) \times \sqrt{2\log(1/u_1)}$

2.9.3 (b) $h(x, y) = (x^2 + y^2,\ x^2 - y^2)$ (c) $h^{-1}(z, w) = (\sqrt{(z + w)/2},\ \sqrt{(z - w)/2})$, at least for $z + w \ge 0$ and $z - w \ge 0$ (d) $f_{Z,W}(z, w) = e^{-\sqrt{(z+w)/2}} / 2\sqrt{z^2 - w^2}$ for $\sqrt{(z + w)/2} \ge 0$ and $1 \le \sqrt{(z - w)/2} \le 4$, i.e., for $z \ge 4$ and $\max(-z,\ z - 64) \le w \le z - 4$, and 0 otherwise

2.9.5 (b) $h(x, y) = (y^4,\ x^4)$ (c) $h^{-1}(z, w) = (w^{1/4},\ z^{1/4})$ (d) $f_{Z,W}(z, w) = e^{-w^{1/4}}$ for $w^{1/4} \geq 0$ and $1 \leq z^{1/4} \leq 4$, i.e., for $w \geq 0$ and $1 \leq z \leq 256$, and 0 otherwise

2.9.7 $p_Z(2) = 1/18$, $p_Z(4) = 1/12$, $p_Z(5) = 1/18$, $p_Z(7) = 1/24$, $p_Z(8) = 1/72$, $p_Z(9) = 1/4$, $p_Z(11) = 3/8$, $p_Z(12) = 1/8$, $p_Z(z) = 0$ otherwise

2.9.9
(a)

(z, w)	(−8,16)	(−7,19)	(−3,11)	(−2,14)	(0,6)	otherwise
$P(Z = z, W = w)$	1/5	1/5	1/5	1/5	1/5	0

(b) $p_Z(z) = 1/5$ for $z = -8, -7, -3, -2, 0$, and otherwise $p_Z(z) = 0$. (c) $p_W(w) = 1/5$ for $w = 6, 11, 14, 16, 19$, and otherwise $p_W(w) = 0$

2.10.1 $Z = -7$ if $U \leq 1/2$, $Z = -2$ if $1/2 < U \leq 5/6$, and $Z = 5$ if $U > 5/6$

2.10.3 $Y \sim$ Exponential(3)

2.10.5 $c_1 = \pm 3\sqrt{2}$ and $c_2 = 5$

2.10.7 (a) For $x < 1$, $F_X(x) = 0$, for $1 \leq x < 2$, $F_X(x) = 1/3$, for $2 \leq x < 4$, $F_X(x) = 1/2$, for $x \geq 4$, $F_X(x) = 1$. (b) The range of t must be restricted on (0, 1] because $F_X^{-1}(0) = -\infty$. $F_X^{-1}(t) = 1$ for $t \in (0, 1/3]$, $F_X^{-1}(t) = 2$ for $t \in (1/3, 1/2]$, and $F_X^{-1}(t) = 4$ for $t \in (1/2, 1]$. (c) For $y < 1$, $F_Y(y) = 0$, for $1 \leq y < 2$, $F_Y(y) = 1/3$, for $2 \leq y < 4$, $F_Y(y) = 1/2$, for $y \geq 4$, $F_Y(y) = 1$.

2.10.9 $Y = F_Z^{-1}(U) = U^{1/4}$

3.1.1 (a) $E(X) = 8/7$ (b) $E(X) = 1$ (c) $E(X) = 8$

3.1.3 (a) $E(X) = -173/12$ (b) $E(Y) = 11$ (c) $E(X^2) = 19$ (d) $E(Y^2) = 370/3$ (e) $E(X^2 + Y^2) = 427/3$ (f) $E(XY - 4Y) = -113/2$

3.1.5 $E(8X - Y + 12) = 8((1 - p)/p) - \lambda + 12$

3.1.7 $E(XY) = 30$

3.1.9 $E(X) = 6$

3.1.11 (a) $E(Z) = 7$ (b) $E(W) = 49/4$

3.1.13 $E(Y) = 7/4$

3.2.1 (a) $C = 1/4, E(X) = 7$ (b) $C = 1/16, E(X) = 169/24$ (c) $C = 5/3093$, $E(X) = -8645/2062$

3.2.3 (a) $E(X) = 17/24$ (b) $E(Y) = 17/8$ (c) $E(X^2) = 11/20$ (d) $E(Y^2) = 99/20$ (e) $E(Y^4) = 216/7$ (f) $E(X^2Y^3) = 27/4$

3.2.5 $E(-5X - 6Y) = -76/3$

3.2.7 $E(Y + Z) = 17/72$

3.2.9 Let $\mu_k = E(X^k)$, then $\mu_1 = 39/25$, $\mu_2 = 64/25$, $\mu_3 = 152/35$

3.2.11 334

3.2.13 $E(Y) = 214.1$

3.2.15 Yes

3.3.1 (a) $\mathrm{Cov}(X, Y) = 2/3$ (b) $\mathrm{Var}(X) = 2$, $\mathrm{Var}(Y) = 32/9$ (c) $\mathrm{Corr}(X, Y) = 1/4$

3.3.3 $\text{Corr}(X, Y) = -0.18292$

3.3.5 $E(XY) = E(X)E(Y)$

3.3.7 (a) $\text{Cov}(X, Z) = 1/9$ (b) $\text{Corr}(X, Z) = 1/\sqrt{46}$

3.3.9 $E\,(X\,(X-1)) = E\left(X^2\right) - E\,(X)$, when $X \sim \text{Binomial}(n, \theta)$, $E\,(X\,(X-1)) = n\,(n-1)\,\theta^2$

3.3.11 $E(X) = E(Y) = 7/2$, $E(XY) = 49/4$, $\text{Cov}(X, Y) = 0$

3.3.13 $\text{Cov}(Z, W) = 0$, $\text{Corr}(Z, W) = 0$

3.3.15 $\text{Cov}(X, Y) = 35/24$

3.4.1 (a) $r_Z(t) = t/(2-t), r'_Z(t) = 2/(2-t)^2, r''_Z(t) = -4/(t-2)^3$ (b) $E(Z) = 2$, $\text{Var}(Z) = 2, m_Z(t) = e^t/(2-e^t), m'_Z(t) = 2e^t/(2-e^t)^2, m''_Z(t) = 2e^t(2+e^t)/(2-e^t)$

3.4.3 $m_Y(s) = e^{\lambda(e^s-1)}$, $m'_Y(s) = \lambda e^s e^{\lambda(e^s-1)}$, so $m'_Y(s) = \lambda$, $m''_Y(s) = (\lambda e^s + \lambda^2 e^{2s})e^{\lambda(e^s-1)}$, so $m''_Y(s) = \lambda + \lambda^2$, $\text{Var}(Y) = \lambda + \lambda^2 - (\lambda)^2 = \lambda$

3.4.5 $m_Y(s) = e^{4s} m_X(3s)$

3.4.7 $m'''_Y(s) = e^{\lambda(e^s-1)}e^s\lambda(1 + 3e^s\lambda + e^{2s}\lambda^2)$, $E(Y^3) = m'''_Y(0) = \lambda(1 + 3\lambda + \lambda^2)$

3.5.1 (a) $E(X \mid Y = 3) = 5/2$ (b) $E(Y \mid X = 3) = 22/3$ (c) $E(X \mid Y = 2) = 5/2$, $E(X \mid Y = 17) = 3$ (d) $E(Y \mid X = 2) = 5/2$, $E(Y \mid X = 3) = 22/3$

3.5.3 (a) $E(Y \mid X = 6) = 25/4$ (b) $E(Y \mid X = -4) = 36/7$ (c) $E(Y \mid X) = 25/4$ whenever $X = 6$, and $E(Y \mid X) = 36/7$ whenever $X = -4$.

3.5.7 $E(Z|W = 4) = 14/3$ (b) $E(W|Z = 4) = 10/3$

3.5.9 (a) $E(X|Y = 0) = 1$ (b) $E(X|Y = 1) = 2$ (c) $E(Y|X = 0) = 0$ (d) $E(Y|X = 1) = 1/3$ (e) $E(Y|X = 2) = 2/3$ (f) $E(Y|X = 3) = 1$ (g) $E(Y|X) = X/3$

3.5.11 (a) $E(X) = 27/19$ (b) $E(Y) = 52/95$ (c) $E(X|Y) = 3(2 + y^3)/4 + 3y^3$ (d) $E(Y|X) = (x^2/2+1/5)/x^2+1/4$ (e) $E[E(Y|X)] = \int_0^2 \frac{3(2+y^3)}{4+3y^3} \cdot \frac{4}{19}(4+3y^3)\,dy = \frac{27}{19}$ (f) $E[E(Y|X)] = \int_0^1 \frac{x^2/2+1/5}{x^2+1/4} \cdot \frac{6}{19}(x^2 + \frac{1}{4})\,dx = \frac{52}{95}$

3.6.1 $3/7$

3.6.3 (a) $1/9$ (b) $1/2$ (c) 2 (d) The upper bound in part (b) is smaller and thus more useful than that in part (c).

3.6.5 $1/4$

3.6.7 (a) 10,000 (b) 12,100

3.6.9 (a) 1 (b) $1/4$

3.6.11 (a) $E(Z) = 8/5$ (b) $32/75$

3.6.13 $7/16$

3.7.1 $E(X_1) = 3$, $E(X_2) = 0$, $E(Y) = 3/5$

3.7.3 $P(X < t) = 0$ for $t < 0$, while $P(X > t) = 1$ for $0 < t < C$ and $P(X > t) = 0$ for $t > C$

3.7.5 $E(X) = 2$

3.7.7 $E(W) = 1/5$

3.7.9 $E(W) = 21/2$

4.1.1 $P(Y_3 = 1) = 1/64$, $P(Y_3 = 3) = 1/64$, $P(Y_3 = 2^{1/3}) = 3/16$, $P(Y_3 = 3^{1/3}) = 3/16$, $P(Y_3 = 4^{1/3}) = 3/32$, $P(Y_3 = 9^{1/3}) = 3/32$, $P(Y_3 = 12^{1/3}) = 3/64$, $P(Y_3 = 18^{1/3}) = 3/64$, $P(Y_3 = 6^{1/3}) = 3/16$

4.1.3 If Z is the sample mean, then $P(Z = 0) = p^2$, $P(Z = 0.5) = 2p(1 - p)$, and $P(Z = 1) = (1 - p)^2$.

4.1.5 For $1 \le j \le 6$, $P(\max = j) = (j/6)^{20} - ((j - 1)/6)^{20}$.

4.1.7 If $W = XY$, then

$$P(W = w) = \begin{cases} 1/36 & \text{if } w = 1, 9, 16, 25, 36, \\ 1/18 & \text{if } w = 2, 3, 5, 8, 10, 15, 18, 20, 24, 30, \\ 1/12 & \text{if } w = 4, \\ 1/9 & \text{if } w = 6, 12, \\ 0 & \text{otherwise.} \end{cases}$$

4.1.9 $p_Y(y) = 1/2$ for $y = 1, 2$; otherwise, $p_Y(y) = 0$

4.2.1 Note that $Z_n = Z$ unless $7 \le U < 7+1/n^2$. Hence, for any $\epsilon > 0$, $P(|Z_n - Z| \ge \epsilon) \le P(7 \le U < 7 + 1/n^2) = 1/5n^2 \to 0$ as $n \to \infty$.

4.2.3 $P(W_1 + \cdots + W_n < n/2) = 1 - P(W_1 + \cdots + W_n \ge n/2) \ge 1 - P(|\frac{1}{n}(W_1 + \cdots + W_n) - \frac{1}{3}| \ge 1/6)$

4.2.5 $P(X_1 + \cdots + X_n > 9n) \le P(|(X_1 + \cdots + X_n)/n - 8| \ge 1)$

4.2.7 For all $\epsilon > 0$ and $n > -2 \ln \epsilon$, $P(|X_n - Y| \ge \epsilon) = P(e^{-H_n} \ge \epsilon) = P(H_n \le -\ln \epsilon) \le P(|H_n - n/2| \ge |n/2 + \ln \epsilon|)$ $n \to \infty$.

4.2.9 By definition, $H_n - 1 \le F_n \le H_n$ and $P(|X_n - Y_n - Z| \ge \epsilon) = P(|H_n - F_n|/(H_n + 1) \ge \epsilon) \le P(1/(H_n + 1) \ge \epsilon) = P(H_n \le (1/\epsilon) - 1) = P(H_n - n/2 \le (1/\epsilon) - 1 - n/2) \le P(|H_n - n/2| \ge |1 + n/2 - 1/\epsilon|)$.

4.2.11 $r = 9/2$

4.3.1 Note that $Z_n = Z$ unless $7 \le U < 7 + 1/n^2$. Also, if $U > 7$, then $Z_n = Z$ whenever $1/n^2 < 7 - U$, i.e., $n > 1/\sqrt{7 - U}$. Hence, $P(Z_n \to Z) \ge P(U \ne 7)$.

4.3.3 $\{(W_1 + \cdots + W_n)/n \to 1/3\} \subseteq \{\exists n;\ (W_1 + \cdots + W_n)/n < 1/2\} = \{\exists n;\ W_1 + \cdots + W_n < n/2\}$

4.3.5 $P(X_n \to X \text{ and } Y_n \to Y) = 1 - P(X_n \not\to X \text{ or } Y_n \not\to Y) \ge 1 - P(X_n \not\to X) - P(Y_n \not\to Y)$

4.3.7 $m = 5$

4.3.9 $r = 9/2$

4.3.11 (a) Suppose there is no such m and from this get a contradiction to the strong law of large numbers. (b) No

4.4.1 $\lim_{n\to\infty} P(X_n = i) = 1/3 = P(X = i)$ for $i = 1, 2, 3$

4.4.3 Here, $P(Z_n \le 1) = 1$, for $0 \le z < 1$, $P(Z_n \le z) = z^{n+1}$, and $P(Z \le z) = 1$ for $z \ge 1$.

4.4.5 $P(S \le 540) \approx \Phi(1/2) = 0.6915$

4.4.7 $P(S \ge 2450) \approx \Phi(-0.51) = 0.3050$

4.4.9 (a) For $0 < y < 1$, $P(Z \le y) = y^2$. (b) For $1 \le m \le n$, $P(X_n \le m/n) = m(m+1)/[n(n+1)]$. (c) For $0 < y < 1$, let $m = \lfloor ny \rfloor$, the biggest integer not greater than ny. Since there is no integer in (m, ny), $P(m/n < X_n < y) \le P(m/n < X_n < (m+1)/n) = 0$. Thus, $P(X_n \le y) = m(m+1)/[n(n+1)]$, where $m = \lfloor ny \rfloor$. (d) For $0 < y < 1$, let $m_n = \lfloor ny \rfloor$, show $m_n/n \to y$ as $n \to \infty$. Then show $P(X_n \le y) \to y^2$ as $n \to \infty$.

4.4.11 $\lambda = 3$

4.4.13 The yearly output, Y, is approximately normally distributed with mean 1300 and variance 433. So, $P(Y < 1280) \approx 0.1685$.

4.5.1 The integral equals $\sqrt{2\pi}\,E(\cos^2(Z))$, where $Z \sim N(0, 1)$.

4.5.3 This integral equals $(1/5)E(e^{-14Z^2})$, where $Z \sim$ Exponential(5).

4.5.5 This sum is approximately equal to $e^5 E(\sin(Z^2))$, where $Z \sim$ Poisson(5).

4.5.7 $(-6.1404, -3.8596)$

4.5.9 $(0.354, 0.447)$

4.5.11 (a) $C = \{\int_0^1 \int_0^1 g(x, y)\,dx\,dy\}^{-1}$ (b) Generate X_i's from $f_X(x) = 3x^2$ for $0 < x < 1$ and Y_i's from $f_Y(y) = 4y^3$ for $0 < y < 1$. Set $D_i = \sin(X_i Y_i)\cos(\sqrt{X_i Y_i}) \times \exp(X_i^2 + Y_i)/12$ and $N_i = X_i \cdot D_i$ for $i = 1, \ldots, n$. 5. Estimate $E(X)$ by $M_n = \overline{N}/\overline{D} = (N_1 + \cdots + N_n)/(D_1 + \cdots + D_n)$.

4.5.13 (a) $J = \int_0^1 \int_0^\infty e^y\, h(x, y)\, I_{[0,\infty)}(y) e^{-y}\, dy\, I_{[0,1]}(x)\, dx$ (b) Generate X_i and Y_i appropriately, set $T_i = e^{Y_i}\, h(X_i, Y_i)$, and estimate J by $M_n = (T_1 + \cdots + T_n)/n$. (c) $J = \int_0^1 \int_0^\infty e^{5y}\, h(x, y)\, I_{[0,\infty)}(y) 5e^{-5y}\, dy\, I_{[0,1]}(x)\, dx$ (d) As in part (b). (e) The estimator having smaller variance is better. So use sample variances to choose between them.

4.6.1 (a) $U \sim N(44,\ 629)$, $V \sim N(-18 - 8C,\ 144 + 25C^2)$ (b) $C = -24/125$

4.6.3 $C_1 = 1/\sqrt{5}$, $C_2 = -3$, $C_3 = 1/\sqrt{2}$, $C_4 = 7$, $C_5 = 2$

4.6.5 Let $Z_1, \ldots, Z_n, W_1, \ldots, W_m \sim N(0, 1)$ be i.i.d. and set $X = (Z_1)^2 + \cdots + (Z_n)^2$ and $Y = (W_1)^2 + \cdots + (W_n)^2$.

4.6.7 $C = \sqrt{n}$

4.6.9 $C_1 = 2/5$, $C_2 = -3$, $C_3 = 2$, $C_4 = 7$, $C_5 = 2$, $C_6 = 1$, $C_7 = 1$

4.6.11 (a) $m = 60$, $K = \sqrt{61}$ (b) $y = 1.671$ (c) $a = 61$, $b = 1$, $c = 60$ (d) $w = 4.00$

5.1.1 The mean survival times for the control group and the treatment group are 93.2 days and 356.2 days, respectively.

5.1.3 For those who are still alive, their survival times will be longer than the recorded values, so these data values are incomplete.

5.1.5 $\bar{x} = -0.1375$

5.1.7 Use the difference $\bar{x} - \bar{y}$.

5.2.1 In Example 5.2.1, the mode is 0. In Example 5.2.2, the mode is 1.

5.2.3 The mixture has density $(5/\sqrt{2\pi})\exp\{-(x+4)^2/2\} + (5/\sqrt{2\pi})\exp\{-(x-4)^2/2\}$ for $-\infty < x < \infty$.

5.2.5 $x = 10$

5.2.7 The mode is $1/3$.

5.2.9 The mode is $x = 0$.

5.3.1 The statistical model for a single response consists of three probability functions $\{\text{Bernoulli}(1/2),\ \text{Bernoulli}(1/3),\ \text{Bernoulli}(2/3)\}$.

5.3.3 The sample $(X_1, \ldots, X_n)$ is a sample from an $N(\mu, \sigma^2)$ distribution, where $\theta = (\mu, \sigma^2) \in \Omega = \{(10, 2), (8, 3)\}$. Both the population mean and variance uniquely identify the population distribution.

5.3.5 A single observation is from an Exponential(θ) distribution, where $\theta \in \Omega = [0, \infty)$. We can parameterize this model by the mean or variance but not by the coefficient of variation.

5.3.7 (a) $\Omega = \{A, B\}$ (b) The value $X = 1$ is observable only when $\theta = A$. (c) Both $\theta = A$ and $\theta = B$ are possible.

5.3.9 P_1

5.4.1

$$F_X(x) = \begin{cases} 0 & x < 1 \\ \frac{4}{10} & 1 \le x < 2 \\ \frac{7}{10} & 2 \le x < 3 \\ \frac{9}{10} & 3 \le x < 4 \\ 1 & 4 \le x \end{cases},\quad f_X(x) = \begin{cases} \frac{4}{10} & x = 1 \\ \frac{3}{10} & x = 2 \\ \frac{2}{10} & x = 3 \\ \frac{1}{10} & x = 4 \end{cases}$$

$\mu_X = \sum_{x=1}^{4} x f_X(x) = 2, \sigma_X^2 = \left(\sum_{x=1}^{4} x^2 f_X(x)\right) - 2^2 = 1$

5.4.3 (a) Yes (b) Use Table D.1 by selecting a row and reading off the first three single numbers (treat 0 in the table as a 10). (c) Using row 108 of Table D.1 (treating 0 as 10): First sample — we obtain random numbers 6, 0, 9, and so compute $(X(\pi_6) + X(\pi_{10}) + X(\pi_9))/3 = 3.0$. Second sample — we obtain random numbers 4, 0, 7, and so compute $(X(\pi_6) + X(\pi_{10}) + X(\pi_9))/3 = 2.6667$. Third sample — we obtain random numbers 2, 0, 2 (note we do not skip the second 2), and so compute $(X(\pi_6) + X(\pi_{10}) + X(\pi_9))/3 = 2.0$.

5.4.5 (c) The shape of a histogram depends on the intervals being used.

5.4.7 It is a categorical variable.

5.4.9 (a) Students are more likely to lie when they have illegally downloaded music so the results of the study will be flawed. (b) Under anonymity, students are more likely to tell the truth, so there will be less error. (c) The probability a student tells the truth is $p = 0.625$. Let Y_i be the answer from student i. Then $(\bar{Y} - (1 - p))/(2p - 1)$ is recorded as an estimate of the proportion of students who have ever downloaded music illegally.

5.5.1 (a) $\hat{f}_X(0) = 0.2667$, $\hat{f}_X(1) = 0.2$, $\hat{f}_X(2) = 0.2667$, $\hat{f}_X(3) = \hat{f}_X(4) = 0.1333$ (b) $\hat{F}_X(0) = 0.2667$, $\hat{F}_X(1) = 0.4667$, $\hat{F}_X(2) = 0.7333$, $\hat{F}_X(3) = 0.8667$, $\hat{F}_X(4) = 1.000$ (d) The mean $\bar{x} = 15$ and the variance $s^2 = 1.952$. (e) The median is 2 and the $IQR = 3$. According to the 1.5 IQR rule, there are no outliers.

5.5.3 (a) $\hat{f}_X(1) = 25/82,\ \hat{f}_X(2) = 35/82,\ \hat{f}_X(3) = 22/82$ (b) No

5.5.5 The sample median is 0, first quartile is -1.150, third quartile is 0.975, and the $IQR = 2.125$. We estimate $F_X(1)$ by $\hat{F}_X(1) = 17/20 = 0.85$.

5.5.7 $\psi(\mu) = \mu + \sigma_0 z_{0.25}$, where $z_{0.25}$ satisfies $\Phi(z_{0.25}) = 0.25$

5.5.9 $\psi(\mu) = \Phi((3-\mu)/\sigma_0)$

5.5.11 $\psi(\mu, \sigma^2) = \Phi((3-\mu)/\sigma)$

5.5.13 $\psi(\theta) = 2\theta(1-\theta)$

5.5.15 $\psi(\theta) = \alpha_0/\beta^2$

6.1.1 The appropriate statistical model is Binomial(n, θ), where $\theta \in \Omega = [0, 1]$ is the probability of having this antibody in the blood. The likelihood function is $L(\theta \mid 3) = \binom{10}{3}\theta^3(1-\theta)^7$.

6.1.3 $L(\theta \mid x_1, \ldots, x_{20}) = \theta^{20}\exp(-(20\bar{x})\theta)$ and $\bar{x}$ is a sufficient statistic.

6.1.5 $c = \binom{10}{4}/\binom{9}{3}$

6.1.7 $L(\theta \mid x_1, ..., x_n) = \prod_{i=1}^n \theta^{x_i}e^{-\theta}/x_i! = \theta^{n\bar{x}}e^{-n\theta}/\prod x_i!$ and $\bar{x}$ is a minimal sufficient statistic.

6.1.9 $L(1|0)/L(2|0) = 4.4817$, the distribution f_1 is 4.4817 times more likely than f_2.

6.1.11 No

6.1.13 No

6.2.1 $\hat{\theta}(1) = a, \hat{\theta}(2) = b, \ \hat{\theta}(3) = b, \hat{\theta}(4) = a$

6.2.3 $\psi(\theta) = \theta^2$ is 1–1 and so $\psi(\hat{\theta}(x_1, ..., x_n)) = \bar{x}^2$ is the MLE.

6.2.5 $\hat{\theta} = \alpha_0/\bar{x}$

6.2.7 $\hat{\alpha} = -n/\sum_{i=1}^n \ln x_i$

6.2.9 $\hat{\alpha} = n/\sum_{i=1}^n \ln(1 + x_i)$

6.2.11 $\hat{\mu}^3 = 32.768 \text{ cm}^3$ is the MLE

6.2.13 A likelihood function can't take negative values.

6.2.15 Equivalent log-likelihood functions differ by an additive constant.

6.3.1 P-value $= 0.592$ and .95-confidence interval is $(4.442, 5.318)$.

6.3.3 P-value $= 0.000$ and .95-confidence interval is $(63.56, 67.94)$.

6.3.5 P-value $= 0.00034$ and .95 confidence interval is $[47.617, 56.383]$. The minimum required sample size is 2.

6.3.7 P-value $= 0.1138$, so not statistically significant and the observed difference of $1.05 - 1 = 0.05$ is well within the range of practical significance.

6.3.9 P-value $= 0.527$

6.3.11 P-value $= 0.014$

6.3.13 (a) $\sum_{i=1}^n (x_i - \bar{x})^2 = \sum_{i=1}^n x_i^2 - n\bar{x}^2$ (b) The plug-in estimator is $\hat{\sigma}^2 = \bar{x}(1-\bar{x})$, so $\hat{\sigma}^2 = s^2(n-1)/n$. (c) bias$(\hat{\sigma}^2) = -\sigma^2/n \to 0$ as $n \to \infty$

6.3.15 (a) Yes (b) No

6.3.17 The P-value 0.22 does not imply the null hypothesis is correct. It may be that we have just not taken a large enough sample size to detect a difference.

6.4.1 $m_3 \pm z_{(1+\gamma)/2} s_3/\sqrt{n} = (26.027, 151.373)$

6.4.3 The method of moments estimator is $\sqrt{m_2 - m_1^2}/m_1$. If $Y = cX$, then $E(Y) = cE(X)$ and $\text{Var}(Y) = c^2\,\text{Var}(X)$.

6.4.5 From the mgf, $m_X'''(0) = 3\sigma^2\mu + \mu^3$. The plug-in estimator is $\hat{\mu}_3 = 3\left(m_2 - m_1^2\right) \times m_1 + m_1^3$, while the method of moments estimator of μ_3 is $m_3 = \frac{1}{n}\sum x_i^3$.

6.4.7 The sample median is estimated by -0.03 and the estimate of the first quartile is -1.28, and for the third quartile is 0.98. Also $\hat{F}(2) = \hat{F}(1.36) = 0.90$.

6.4.9 The bootstrap procedure is sampling from a discrete distribution and by the CLT the distribution of the bootstrap mean is approximately normal when n and m are large. The delta theorem justifies the approximate normality of functions of the bootstrap mean under conditions.

6.4.11 The maximum number of possible values is $1 + \binom{n}{2} = 1 + n(n-1)/2$. Here, 0 is obtained when $i = j$. The bootstrap sample range $y_{(n)} - y_{(1)}$ has the largest possible value $x_{(n)} - x_{(1)}$ and smallest possible value of 0. If there are many repeated x_i values in the bootstrap sample, then the value 0 will occur with high probability for $y_{(n)} - y_{(1)}$ and so the bootstrap distribution of the sample range will not be approximately normal.

6.5.1 $n/2\sigma^4$

6.5.3 n/α^2

6.5.5 $2/\bar{x} \pm (2/\bar{x})\, z_{0.95}/\sqrt{2n} = (9.5413 \times 10^{-4}, 1.5045 \times 10^{-3})$

6.5.7 $\hat{\alpha} \pm (\hat{\alpha}/\sqrt{n}) z_{(1+\gamma)/2} = (0.18123, 0.46403)$ as a 0.95-confidence interval, and this does not contain $\alpha = 1 + 1/25 = 1.04$.

6.5.9 $[0, \min((1+\bar{x})^{-1} + n^{-1/2}(1+\bar{x})^{-1}\sqrt{\bar{x}(1+\bar{x})^{-1}} z_\gamma, 1)]$

7.1.1

θ	1	2	3
$\pi(\theta \mid s = 1)$	3/16	1/4	9/16
$\pi(\theta \mid s = 2)$	3/14	4/7	3/14

7.1.3 The prior probability that θ is positive is 0.5, and the posterior probability is 0.9992.

7.1.5 $\theta^{\alpha-n-1} e^{-\beta\theta} I_{[x_{(n)},\infty)}(\theta) \,/\, \int_{x_{(n)}}^{\infty} \theta^{\alpha-n-1} e^{-\beta\theta}\, d\theta$

7.1.7 $\mu \mid \sigma^2, x_1, \ldots, x_n \sim N(5.5353, \frac{4}{81}\sigma^2)$ and $1/\sigma^2 \mid x_1, \ldots, x_n \sim \text{Gamma}(11, 41.737)$

7.1.9 (a) $(n+1)\theta^n I_{[0.4,0.6]}(\theta)/(0.6^{n+1} - 0.4^{n+1})$ (b) No (c) The prior must be greater than 0 on any parameter values that we believe are possible.

7.1.11 (a) $\Pi(|\theta| = 0) = 1/6$, $\Pi(|\theta| = 1) = 1/3$, $\Pi(|\theta| = 2) = 1/3$, $\Pi(|\theta| = 3) = 1/6$, so $|\theta|$ is not uniformly distributed on $\{0, 1, 2, 3\}$. (b) No

7.2.1 $\frac{\Gamma(\alpha+\beta+n)\Gamma(n\bar{x}+\alpha+m)}{\Gamma(n\bar{x}+\alpha)\Gamma(\alpha+\beta+n+m)}$

7.2.3 $E\left(1/\sigma^2 \mid x_1, \ldots, x_n\right) = (\alpha_0 + n/2)/\beta_x$, and the posterior mode is $1/\hat{\sigma}^2 = (\alpha_0 + n/2 - 1)/\beta_x$.

7.2.5 As in Example 7.2.4 the posterior distribution of θ_1 is Beta$(f_1+\alpha_1,\ f_2+\cdots+f_k+\alpha_2+\cdots+\alpha_k)$ so $E(\theta_1 \mid x_1, \ldots, x_n) = \Gamma(n+\sum_{i=1}^k \alpha_i)\Gamma(f_1+\alpha_1+1)\,/\,\Gamma(f_1+\alpha_1)\,\Gamma(n$

$+\sum_{i=1}^{k}\alpha_i+1)$ and maximizes $\ln((\theta_1)^{f_1+\alpha_1-1}(1-\theta_1)^{\sum_{i=2}^{k}(f_i+\alpha_i)-1})$ for the posterior mode.

7.2.7 Recall that the posterior distribution of θ_1 in Example 7.2.2 is Beta$(f_1+\alpha_1,\ f_2+\cdots+f_k+\alpha_2+\cdots+\alpha_k)$. Find the second moment and use $\text{Var}(\theta_1\,|\,x_1,...,x_n) = E\left(\theta_1^2\,|\,x_1,...,x_n\right)-(E\,(\theta_1\,|\,x_1,...,x_n))^2$. Now $0\le f_1/n\le 1$, so $\text{Var}(\theta_1\,|\,x_1,...,x_n) = (f_1/n+\alpha_1)\,(\sum_{i=2}^{k}(f_i/n+\alpha_i))/\{n(1+\sum_{i=1}^{k}\alpha_i/n+1/n)(1+\sum_{i=1}^{k}\alpha_i/n)^2\}\to 0$ as $n\to\infty$.

7.2.9 The posterior predictive density of x_{n+1} is obtained by averaging the $N(\bar{x},\,(1/\tau_0^2+n/\sigma_0^2)^{-1}\sigma_0^2)$ density with respect to the posterior density of μ, so we must have that this is also the posterior predictive distribution.

7.2.11 The posterior predictive distribution of $t=x_{n+1}$ is $(n\bar{x}+\beta_0)$Pareto$(n+\alpha_0)$. So the posterior mode is $\hat{t}=0$, the posterior expectation is $(n\bar{x}+\beta_0)/(n+\alpha_0-1)$, and the posterior variance is $(n\bar{x}+\beta_0)^2(n+\alpha_0)/[(n+\alpha_0-1)^2(n+\alpha_0-2)]$.

7.2.13 (a) The posterior distribution of σ^2 is inverse Gamma$(n/2+\alpha_0,\beta_x)$, where $\beta_x=(n-1)s^2/2+n(\bar{x}-\mu_0)^2/2+\beta_0$. (b) $E(\sigma^2\,|\,x_1,...,x_n)=\beta_x/(n/2+\alpha_0-1)$. (c) To assess the hypothesis $H_0:\sigma^2\le\sigma_0^2$, compute the probability $\Pi(1/\sigma^2\ge 1/\sigma_0^2\,|\,x_1,..,x_n)=1-G(2\beta_x/\sigma_0^2;2\alpha_0+n)$ where $G(\cdot\,;2\alpha_0+n)$ is the $\chi^2(2\alpha_0+n)$ cdf.

7.2.15 (a) The odds in favor of $A=1/$odds in favor of A^c. (b) $BF(A)=1/BF(A^c)$

7.2.17 Statistician I's posterior probability for H_0 is 0.0099. Statistician II's posterior probability for H_0 is 0.0292. Hence, Statistician II has the bigger posterior belief in H_0.

7.2.19 The range of a Bayes factor in favor of A ranges in $[0,\infty)$. If A has posterior probability equal to 0, then the Bayes factor will be 0.

7.3.1 (3.2052, 4.4448)

7.3.3 The posterior mode is $\hat{\mu}=(nt/v_0^2+\mu_0/\sigma_0^2)/(n/v_0^2+1/\sigma_0^2)$ and $\hat{\sigma}^2(x_1,\ldots,x_n)=(n/v_0^2+1/\sigma_0^2)^{-1}$. Hence, the asymptotic γ-credible interval is $(\hat{\mu}-z_{(1+\gamma)/2}\hat{\sigma},\,\hat{\mu}+z_{(1+\gamma)/2}\hat{\sigma})$.

7.3.5 For a sample $(x_1,\ldots,x_n)$ the posterior distribution is $N(\bar{x},1/n)$ restricted to $[0,1]$. A simple Monte Carlo algorithm for the posterior distribution is 1. Generate η from $N(\bar{x},1/n)$, 2. Accept η if it is in $[0,1]$ and return to step 1 otherwise. If the true value θ_* is not in $[0,1]$, then the acceptance rate will be very small for large n.

7.4.1 The posterior density is proportional to $\lambda^{n+\alpha-1}\exp\{-\lambda(\ln\left(\prod(1+x_i)\right)+\beta)\}$.

7.4.3 (a) The maximum value of the prior predictive is obtained when $\tau=1$. (b) The posterior of θ given $\tau=1$ is

$$\pi_1(\theta\,|\,1,1,3)=\begin{cases}\frac{(1/2)(1/3)^3}{59/1728}=\frac{32}{59} & \theta=a\\ \frac{(1/2)(1/2)^2(1/8)}{59/1728}=\frac{27}{59} & \theta=b.\end{cases}$$

7.4.5 The prior predictive is given by $m_{\alpha,\beta}(x_1,...,x_n)=\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\frac{\Gamma(\alpha+n\bar{x})\Gamma(\beta+n(1-\bar{x}))}{\Gamma(\alpha+\beta+n)}$. Based on the prior predictive, we would select the prior given by $\alpha=1,\beta=1$.

7.4.7 Jeffreys' prior is $\sqrt{n}\theta^{-1/2}(1-\theta)^{-1/2}$. The posterior distribution of θ is Beta$(n\bar{x}$ $+1/2, n(1-\bar{x})+1/2)$.

7.4.9 The prior distribution is $\theta \sim N(66, \sigma^2)$ with $\sigma^2 = 101.86$.

7.4.11 Let the prior be $\theta \sim$ Exponential(λ) with $\lambda = 0.092103$.

8.1.1 $L(1\,|\,\cdot) = (3/2)\,L(2\,|\,\cdot)$, so by Section 6.1.1 T is a sufficient statistic. The conditional distributions of s are as follows.

	$s=1$	$s=2$	$s=3$	$s=4$
$f_a(s\,|\,T=1)$	$\frac{1/3}{1/3+1/6}=\frac{2}{3}$	$\frac{1/6}{1/3+1/6}=\frac{1}{3}$	0	0
$f_b(s\,|\,T=1)$	$\frac{1/2}{1/2+1/4}=\frac{2}{3}$	$\frac{1/4}{1/2+1/4}=\frac{1}{3}$	0	0

	$s=1$	$s=2$	$s=3$	$s=4$
$f_a(s\,|\,T=3)$	0	0	1	0
$f_b(s\,|\,T=3)$	0	0	1	0

	$s=1$	$s=2$	$s=3$	$s=4$
$f_a(s\,|\,T=4)$	0	0	0	1
$f_b(s\,|\,T=4)$	0	0	0	1

8.1.3 $\bar{x}^2 + (1-1/n)\,\sigma_0^2$

8.1.5 UMVU for $5+2\mu$

8.1.7 $\bar{x}/\alpha_0$

8.1.9 $n^{-1}\sum_{i=1}^{n} I_{(-1,1)}(X_i)$

8.1.11 Yes

8.2.1 When $\alpha = 0.1$, $c_0 = 3/2$ and $\gamma = ((1/10)-(1/12))\,/\,(1/2) = 1/30$. The power of the test is $23/120$. When $\alpha = 0.05$, $c_0 = 2$ and $\gamma = ((1/20)-0)\,/\,(1/12) = 3/5$.The power of the test is $1/10$.

8.2.3 By (8.2.6) the optimal 0.01 test is of the form

$$\varphi_0(\bar{x}) = \begin{cases} 1 & \bar{x} \ge 1 + \frac{\sqrt{2}}{\sqrt{10}}2.3263 \\ 0 & \bar{x} < 1 + \frac{\sqrt{2}}{\sqrt{10}}2.3263 \end{cases} = \begin{cases} 1 & \bar{x} \ge 2.0404 \\ 0 & \bar{x} < 2.0404. \end{cases}$$

8.2.5 (a) 0 (b) Suppose $\theta > 1$. The power function is $\beta(\theta) = 1 - 1/\theta$.

8.2.7 $n \ge 4$

8.2.9 The graph of the power function of the UMP size α test function lies above the graph of the power function of any other size test α function.

8.3.1 $\Pi(\theta = 1\,|\,2) = 2/5$, $\Pi(\theta = 2\,|\,2) = 3/5$, so $\Pi(\theta = 2\,|\,2) > \Pi(\theta = 1\,|\,2)$ and we accept $H_0 : \theta = 2$.

8.3.3 The Bayes rule is given by $(1/\tau_0^2 + n/\sigma_0^2)^{-1}(\mu_0/\tau_0^2 + n\bar{x}/\sigma_0^2)$, which converges to $\bar{x}$ as $\tau_0 \to \infty$.

8.3.5 The Bayes rule is given by $(n\alpha_0 + \tau_0)\,/\,(n\bar{x} + \upsilon_0)$ and by the weak law of large numbers this converges in probability to β as $n \to \infty$.

8.3.7 The Bayes rule rejects whenever

$$BF_{H_0} = \frac{\exp\left(-\frac{n}{2\sigma_0^2}\left(\bar{x}-\mu_0\right)^2\right)}{\tau_0^{-1}\left(\frac{n}{\sigma_0^2}+\frac{1}{\tau_0^2}\right)^{-1/2}\exp\left(\frac{1}{2}\left(\frac{1}{\tau_0^2}+\frac{n}{\sigma_0^2}\right)^{-1}\left(\frac{\mu_0}{\tau_0^2}+\frac{n}{\sigma_0^2}\bar{x}\right)^2-\frac{1}{2}\left(\frac{\mu_0^2}{\tau_0^2}+\frac{n\bar{x}^2}{\sigma_0^2}\right)\right)}$$

is less than $(1-p_0)/p_0$. As $\tau_0^2 \to \infty$, the denominator converges to 0, so in the limit we never reject H_0.

8.4.1 The model is given by the collection of probability functions $\{\theta^{n\bar{x}}(1-\theta)^{n-n\bar{x}} : \theta \in [0,1]\}$ on the set of all sequences $(x_1,\ldots,x_n)$ of 0's and 1's. The action space is $\mathsf{A} = [0,1]$, the correct action function is $A(\theta) = \theta$, and the loss function is $L(\theta, a) = (\theta - a)^2$. The risk function for T is $R_T(\theta) = \mathrm{Var}_\theta(\bar{x}) = \theta(1-\theta)/n$.

8.4.3 The model is the set of densities $\{(2\pi\sigma_0^2)^{-1/2}\exp\{-\sum_{i=1}^n (x_i-\mu)^2/2\sigma_0^2\} : \mu \in R^1\}$ on R^n. The action space is $\mathsf{A} = R^1$, the correct action function is $A(\mu) = \mu$, and the loss function is $L(\mu, a) = (\mu - a)^2$. The risk function for T is $R_T(\mu) = \mathrm{Var}_\mu(\bar{x}) = \sigma_0^2/n$.

8.4.5 (a) $R_d(a) = 1/2$, $R_d(b) = 3/4$ (b) No. Consider the risk function of the decision function d^* given by $d^*(1) = b, d^*(2) = a, d^*(3) = b, d^*(4) = a$.

9.1.1 The observed discrepancy statistic is given by $D(r) = 22.761$ and the P-value is $P(D(R) > 22.761) = 0.248$, which doesn't suggest evidence against the model.

9.1.3 (c) The plots suggest that the normal assumption seems reasonable.

9.1.5 The observed counts are given in the following table.

Interval	Count
(0.0, 0.2]	4
(0.2, 0.4]	7
(0.4, 0.6]	3
(0.6, 0.8]	4
(0.8, 1]	2

The Chi-squared statistic is equal to 3.50 and the P-value is given by ($X^2 \sim \chi^2(4)$) $P\left(X^2 \geq 3.5\right) = 0.4779$. Therefore, we have no evidence against the Uniform model being correct.

9.1.7 (a) The probability of the event $s = 3$ is 0 based on the probability measure P having S as its support. The most appropriate P-value is 0. (b) 0.729

9.1.9 No

9.1.11 (a) The conditional probability function of $(x_1,\ldots,x_n)$ is

$$\theta^{n\bar{x}}(1-\theta)^{n(1-\bar{x})}/\binom{n}{n\bar{x}}\theta^{n\bar{x}}(1-\theta)^{n(1-\bar{x})} = 1/\binom{n}{n\bar{x}}.$$

(b) Hypergeometric$(n, \lfloor n/2 \rfloor, n\bar{x}_0)$ (c) 0.0476

9.2.1 (a) No (b) P-value is $1/10$, so there is little evidence of a prior–data conflict. (c) P-value is $1/300$, so there is some evidence of a prior–data conflict.

9.2.3 We can write $\bar{x} = \mu + z$, where $z \sim N(0, \sigma_0^2/n)$ is independent of $\mu \sim N(\mu_0, \tau_0^2)$.

9.2.5 The P-value for checking prior–data conflict is 0. Hence, there is definitely a prior–data conflict.

10.1.1 For any x_1, x_2 (that occur with positive probability) and y, we have $P(Y = y \mid X = x_1) = P(Y = y \mid X = x_2)$. Thus $P(X = x_1, Y = y) = P(X = x_2, Y = y)P(X = x_1)/P(X = x_2)$, and summing this over x_1 leads to $P(X = x_2, Y = y) = P(X = x_2)P(Y = y)$. For the converse, show $P(Y = y \mid X = x) = P(Y = y)$.

10.1.3 X and Y are related.

10.1.5 The conditional distributions $P(Y = y \mid X = x)$ will change with x whenever X is not degenerate.

10.1.7 If the conditional distribution of life-length given various smoking habits changes, then we can conclude that these two variables are related, but we cannot conclude that this relationship is a cause–effect relationship due to the possible existence of confounding variables.

10.1.9 The researcher should draw a random sample from the population of voters and ask them to measure their attitude toward a particular political party on a scale from favorably disposed to unfavorably disposed. Then the researcher should randomly select half of this sample to be exposed to a negative ad, while the other half is exposed to a positive ad. They should all then be asked to measure their attitude toward the particular political party on the same scale. Next compare the conditional distribution of the response variable Y (the change in attitude from before seeing the ad to after) given the predictor X (type of ad exposed to), using the samples to make inference about these distributions.

10.1.11 (a) $\{(0, 100), (1, 100)\}$ (b) A sample has not been taken from the population of interest. The individuals involved in the study have volunteered and, as a group, they might be very different from the full population. (c) We should group the individuals according to their initial weight W into homogenous groups (blocks) and then randomly apply the treatments to the individuals in each block.

10.1.13 (a) The response variable could be the number of times an individual has watched the program. A suitable predictor variable is whether or not they received the brochure. (b) Yes, as we have controlled the assignment of the predictor variable.

10.1.15 W has a relationship with Y and X has a relationship with Y.

10.1.17 (a)

	$X = 0$	$X = 1$	Sum
Rel. Freq.	0.5	0.5	1.0

(b)

	$Y = 0$	$Y = 1$	Sum
Rel. Freq.	0.7	0.3	1.0

(c)

Rel. Freq.	$X = 0$	$X = 1$	Sum
$Y = 0$	0.3	0.4	0.7
$Y = 1$	0.2	0.1	0.3
sum	0.5	0.5	1.0

(d)

$P(Y = y \mid X = x)$	$y = 0$	$y = 1$	sum
$x = 0$	0.6	0.4	1.0
$x = 1$	0.8	0.2	1.0

(e) Yes

10.1.19 X and Y are related. We see that only the variance of the conditional distribution changes as we change X.

10.1.21 The correlation is 0, but X and Y are related.

10.2.1 The chi-squared statistic is equal to $X_0^2 = 5.7143$ and, with $X^2 \sim \chi^2(2)$, the P-value equals $P(X^2 > 5.7143) = 0.05743$. Therefore, we don't have evidence against the null hypothesis of no difference in the distributions of thunderstorms between the two years, at least at the 0.05 level.

10.2.3 The chi-squared statistic is equal to $X_0^2 = 0.10409$ and, with $X^2 \sim \chi^2(1)$, the P-value equals $P(X^2 > 4.8105) = 0.74698$. Therefore, we have no evidence against the null hypothesis of no relationship between the two digits.

10.2.5 (a) The chi-squared statistic is equal to $X_0^2 = 10.4674$ and, with $X^2 \sim \chi^2(4)$, the P-value equals $P(X^2 > 10.4674) = 0.03325$. Therefore, we have some evidence against the null hypothesis of no relationship between hair color and gender. (c) The standardized residuals are given in the following table. They all look reasonable, so nothing stands out as an explanation of why the model of independence does not fit. Overall, it looks like a large sample size has detected a small difference.

	$Y =$ fair	$Y =$ red	$Y =$ medium	$Y =$ dark	$Y =$ jet black
$X =$ m	−1.07303	0.20785	1.05934	−0.63250	1.73407
$X =$ f	1.16452	−0.22557	−1.14966	0.68642	−1.88191

10.2.7 We should first generate a value for $X_1 \sim$ Dirichlet$(1, 3)$. Then generate U_2 from the Beta$(1, 2)$ distribution and set $X_2 = (1 - X_1)\, U_2$. Next generate U_3 from the Beta$(1, 1)$ distribution and set $X_3 = (1 - X_1 - X_2)\, U_3$. Finally set $X_4 = 1 - X_1 - X_2 - X_3$.

10.2.9 Then there are 36 possible pairs (i, j) for $i, j = 1, \ldots, 6$. Let f_{ij} denote the frequency for (i, j) and compute chi-squared statistic, $X^2 = \sum_{i=1}^{6} \sum_{j=1}^{6} (f_{ij} - f_{i\cdot}\, f_{\cdot j}/n)^2 / (f_{i\cdot}\, f_{\cdot j}/n)$. Compute the P-value $P(\chi^2(25) > X^2)$.

10.2.11 We look at the differences $\left| f_{ij} - f\, i\cdot f_{\cdot j}/n \right|$ to see how big these are.

10.3.1 $\bar{x}$

10.3.3 $\bar{x}$

10.3.5 (b) $y = 29.9991 + 2.10236x$ (e) The plot of the standardized residuals against X indicates very clearly that there is a problem with this model. (f) Based on part (e), it is not appropriate to calculate confidence intervals for the intercept and slope. (g) Nothing can be concluded about the relationship between Y and X based on this model, as we have determined that it is inappropriate. (h) $R^2 = 486.193/7842.01 = 0.062$, which is very low.

10.3.7 (b) $b_2 = 1.9860$ and $b_1 = 58.9090$
(d) The standardized residual of the ninth week departs from the other residuals in part (c). This provides some evidence that the model is not correct. (e) The confidence interval for β_1 is [44.0545, 72.1283], and the confidence interval for β_2 is [0.0787, 3.8933]. (f) The ANOVA table is as follows.

Source	Df	Sum of Squares	Mean Square
X	1	564.0280	564.0280
Error	10	1047.9720	104.7972
Total	11	1612.0000	

So the F-statistic is $F = 5.3821$ and $P(F(1, 10) \geq 5.3821) < 0.05$ from Table D.5. Hence, we conclude there is evidence against the null hypothesis of no linear relationship between the response and the predictor. (g) $R^2 = 0.3499$ so, almost 35% of the observed variation in the response is explained by changes in the predictor.

10.3.9 In general, $E(Y \mid X) = \exp(\beta_1 + \beta_2 X)$ is not a simple linear regression model since it cannot be written in the form $E(Y \mid X) = \beta_1^* + \beta_2^* V$, where V is an observed variable and the β_i^* are unobserved parameter values.

10.3.11 We can write $E(Y \mid X) = E(Y \mid X^2)$ in this case and $E(Y \mid X^2) = \beta_1 + \beta_2 X^2$, so this is a simple linear regression model but the predictor is X^2 not X.

10.3.13 $R^2 = 0.05$ indicates that the linear model explains only 5% of the variation in the response, so the model will not have much predictive power.

10.4.1 (b) Both plots look reasonable, indicating no serious concerns about the correctness of the model assumptions. (c) The ANOVA table for testing $H_0 : \beta_1 = \beta_2 = \beta_3$ is given below.

Source	Df	SS	MS
A	2	4.37	2.18
Error	9	18.85	2.09
Total	11	23.22	

The F statistic for testing H_0 is given by $F = 2.18/2.09 = 1.0431$, with P-value $P(F > 1.0431) = 0.39135$. Therefore, we don't have evidence against the null hypothesis of no difference among the conditional means of Y given X. (d) Since we did not find any relationship between Y and X, there is no need to calculate these confidence intervals.

10.4.3 (b) Both plots indicate a possible problem with the model assumptions. (c) The ANOVA table for testing $H_0 : \beta_1 = \beta_2$ is given below.

Source	Df	SS	MS
Cheese	1	0.114	0.114
Error	10	26.865	2.686
Total	11	26.979	

The F statistic for testing H_0 is given by $F = 0.114/2.686 = .04$ and with the P-value $P(F > .04) = 0.841$. Therefore, we do not have any evidence against the null hypothesis of no difference among the conditional means of Y given Cheese.

10.4.5 (b) Both plots look reasonable, indicating no concerns about the correctness of the model assumptions. (c) The ANOVA table for testing $H_0 : \beta_1 = \beta_2 = \beta_3 = \beta_4$ follows.

Source	Df	SS	MS
Treatment	3	19.241	6.414
Error	20	11.788	0.589
Total	23	31.030	

The F statistic for testing H_0 is given by $F = 6.414/0.589 = 10.89$ and with P-value $P(F > 10.89) = 0.00019$. Therefore, we have strong evidence against the null hypothesis of no difference among the conditional means of Y, given the predictor. (d) The 0.95-confidence intervals for the difference between the means are given in the following table.

```
Family error rate = 0.192
Individual error rate = 0.0500

Critical value = 2.086

Intervals for (column level mean) - (row level mean)

                1          2          3

     2    -0.3913
           1.4580

     3    -2.2746    -2.8080
          -0.4254    -0.9587

     4    -2.5246    -3.0580    -1.1746
          -0.6754    -1.2087     0.6746
```

10.4.7 (b) Treating the marks as separate samples, the ANOVA table for testing any difference between the mean mark in Calculus and the mean mark in Statistics follows.

Source	Df	SS	MS
Course	1	36.45	36.45
Error	18	685.30	38.07
Total	19	721.75	

The F statistic for testing $H_0 : \beta_1 = \beta_2$ is given by $F = 36.45/38.07 = 0.95745$, with the P-value equal to $P(F > 0.95745) = 0.3408$. Therefore, we do not have any evidence against the null hypothesis of no difference among the conditional means of Y given Course.

Both residual plots look reasonable, indicating no concerns about the correctness of the model assumptions. (c) Treating these data as repeated measures, the mean difference between the mark in Calculus and the mark in Statistics is given by $\bar{d} = -2.7$ with standard deviation $s = 2.00250$. The P-value for testing $H_0 : \mu_1 = \mu_2$, is 0.0021, so we have strong evidence against the null. Hence, we conclude that there is a difference between the mean mark in Calculus and the mean mark in Statistics. A normal probability plot of the data does not indicate any reason to doubt model assumptions. (d) $r_{xy} = 0.944155$

10.4.9 When Y_1 and Y_2 are measured on the same individual, we have that $\text{Var}(Y_1 - Y_2) = 2(\text{Var}(Y_1) - \text{Cov}(Y_1, Y_2)) > 2\text{Var}(Y_1)$ since $\text{Cov}(Y_1, Y_2) < 0$. If we had measured Y_1 and Y_2 on independently randomly selected individuals, then we would have that $\text{Var}(Y_1 - Y_2) = 2\text{Var}(Y_1)$.

10.4.11 The difference of the two responses Y_1 and Y_2 is normally distributed, i.e., $Y_1 - Y_2 \sim N(\mu, \sigma^2)$.

10.4.13 (1) The conditional distribution of Y given (X_1, X_2), depends on (X_1, X_2) only through $E(Y \mid X_1, X_2)$, and the error $Z = Y - E(Y \mid X_1, X_2)$ is independent of (X_1, X_2). (2) The error $Z = Y - E(Y \mid X_1, X_2)$ is normally distributed. (3) X_1 and X_2 do not interact.

10.5.1 $F(x) = \int_{-\infty}^{x} e^{-t}\left(1 + e^{-t}\right)^{-2} dt = \left(1 + e^{-t}\right)^{-1} |_{-\infty}^{x} = \left(1 + e^{-x}\right)^{-1} \to 1$ as $x \to \infty$ and $p = F(x)$, implies $x = \ln(p/(1-p))$.

10.5.3 Let $l = l(p) = \ln(p/(1-p))$ be the log odds so $e^l = p/(1-p) = 1/(1/p - 1)$. Hence, $e^l/(1 + e^l) = p$, and substitute $l = \beta_1 + \beta_2 x$.

10.5.5 $P(Y = 1|X_1 = x_1, \ldots, X_k = x_k) = 1/2 + \arctan(\beta_1 x_1 + \cdots + \beta_k x_k)/\pi$

11.1.1 (a) 0 (b) 0 (c) $1/3$ (d) $2/3$ (e) 0 (f) $4/9$ (g) 0 (h) $1/9$ (i) 0 (j) 0 (k) 0.00925 (l) 0 (m) 0.0987

11.1.3 (a) $5/108$ (b) $5/216$ (c) $5/72$ (d) By the law of total probability, $P(X_3 = 8) = P(X_1 = 6,\ X_3 = 8) + P(X_1 = 8,\ X_3 = 8)$.

11.1.5 (a) Here, $P(\tau_c < \tau_0) \doteq 0.89819$. That is, if you start with \$9 and repeatedly make \$1 bets having probability 0.499 of winning each bet, then the probability you will reach \$10 before going broke is equal to 0.89819 (b) 0.881065 (c) 0.664169 (d) 0.0183155 (e) 4×10^{-18} (f) 2×10^{-174}

11.1.7 We use Theorem 11.1.1. (a) $1/4$ (b) $3/4$ (c) 0.0625 (d) $1/4$ (e) 0 (f) 1 (g) We know that the initial fortune is 5, so to get to 7 in two steps, the walk must have been at 6 after the first step.

11.1.9 (a) $18/38$ (b) 0.72299 (c) 0.46056 (d) 0 (e) In the long run, the gambler loses money.

11.2.1 (a) 0.7 (b) 0.1 (c) 0.2 (d) $1/4$ (e) $1/4$ (f) $1/2$ (g) 0.3

11.2.3 (a) $P_0(X_2 = 0) = 0.28$, $P_0(X_2 = 1) = 0.72$, $P_1(X_2 = 0) = 0.27$, $P_1(X_2 = 1) = 0.73$ (b) $P_0(X_3 = 1) = 0.728$

11.2.5 (a) $1/2$ (b) 0 (c) $1/2$ (d) $1/2$ (e) $1/10$ (f) $2/5$ (g) $37/100$ (h) $11/20$ (i) 0 (j) 0 (k) 0 (l) 0 (m) No

11.2.7 This chain is doubly stochastic, i.e., has $\sum_i p_{ij} = 1$ for all j. Hence, we must have the uniform distribution ($\pi_1 = \pi_2 = \pi_3 = \pi_4 = 1/4$) as a stationary distribution.

11.2.9 (a) By either increasing or decreasing one step at a time, we see that for all i and j, we have $p_{ij}^{(n)} > 0$ for some $n \le d$. (b) Each state has period 2. (c) If i and j are two or more apart, then $p_{ij} = p_{ji} = 0$. If $j = i + 1$, then $\pi_i p_{ij} = (1/2^d)((d-1)!/i!(d-i-1)!)$, while $\pi_j p_{ji} = (1/2^d)((d-1)!/i!(d-i-1)!)$.

11.2.11 (a) This chain is irreducible. (b) The chain is aperiodic. (c) $\pi_1 = 2/9, \pi_2 = 3/9, \pi_3 = 4/9$ (d) $\lim_{n\to\infty} P_1(X_n = 2) = \pi_2 = 3/9 = 1/3$, so $P_1(X_{500} = 2) \approx 1/3$.

11.2.13 $P_1(X_1 + X_2 \ge 5) = 0.54$

11.2.15 (a) The chain is irreducible. (b) The chain is not aperiodic.

11.3.1 First, choose any initial value X_0. Then, given $X_n = i$, let $Y_{n+1} = i+1$ or $i-1$, with probability $1/2$ each. Let $j = Y_{n+1}$ and let $\alpha_{ij} = \min(1,\ e^{-(j-13)^4+(i-13)^4})$. Then let $X_{n+1} = j$ with probability α_{ij}, otherwise let $X_{n+1} = i$ with probability $1 - \alpha_{ij}$.

11.3.3 First, choose any initial value X_0. Then, given $X_n = i$, let $Y_{n+1} = i + 1$ with probability $7/9$ or $Y_{n+1} = i - 1$ with probability $2/9$. Let $j = Y_{n+1}$ and, if $j = i + 1$, let $\alpha_{ij} = \min(1,\ e^{-j^4-j^6-j^8}(2/9)/e^{-i^4-i^6-i^8}(7/9))$ or, if $j = i - 1$, then let $\alpha_{ij} = \min(1,\ e^{-j^4-j^6-j^8}(7/9)/e^{-i^4-i^6-i^8}(2/9))$. Then let $X_{n+1} = j$ with probability α_{ij}, otherwise let $X_{n+1} = i$ with probability $1 - \alpha_{ij}$.

11.3.5 Let $\{Z_n\}$ be i.i.d. $\sim N(0, 1)$. First, choose any initial value X_0. Then, given $X_n = x$, let $Y_{n+1} = X_n + \sqrt{10}\, Z_{n+1}$. Let $y = Y_{n+1}$ and let $\alpha_{xy} = \min(1,\ \exp\{-y^4 - y^6 - y^8 + x^4 + x^6 + x^8\})$. Then let $X_{n+1} = y$ with probability α_{xy}, otherwise let $X_{n+1} = x$ with probability $1 - \alpha_{xy}$.

11.4.1 $C = 12/5$

11.4.3 $p = 1/3$

11.4.5 $P(X_n = 4) = 5/8$

11.4.7 (a) Here, $E(X_{n+1}\,|\,X_n) = (1/4)(3X_n) + (3/4)(X_n/3) = X_n$. (b) T is nonnegative, integer-valued, and does not look into the future, so it is a stopping time. (c) $E(X_T) = X_0 = 27$. (d) $P(X_T = 1) = 27/40$

11.5.1 (a) $1/2$ (b) 0 (c) $1/4$ (d) We have $P(Y_1^{(M)} \ge 1) = P(Y_{M/M}^{(M)} \ge \sqrt{M}/\sqrt{M})$. Hence, $P(Y_1^{(1)} \ge 1) = 1/2$, $P(Y_1^{(2)} \ge 1) = 1/4$, $P(Y_1^{(3)} \ge 1) = 3/8$, $P(Y_1^{(4)} \ge 1) = 5/16$.

11.5.3 (a) $P(B_2 \ge 1) = \Phi(-1/\sqrt{2})$ (b) $P(B_3 \le -4) = \Phi(-4/\sqrt{3})$ (c) $P(B_9 - B_5 \le 2.4) = 1 - \Phi(-2.4/2)$ (d) $P(B_{26} - B_{11} > 9.8) = \Phi(-9.8/\sqrt{15})$ (e) $P(B_{26.3} \le -6) = \Phi(-6/\sqrt{26.3})$ (f) $P(B_{26.3} \le 0) = \Phi(0/\sqrt{26.3}) = 1/2$

11.5.5 $E(B_{13}B_8) = 8$

11.5.7 (a) $3/4$ (b) $1/4$ (c) The answer in part (a) is larger because -5 is closer to $B_0 = 0$ than 15 is, whereas -15 is farther than 5 is. (d) $1/4$ (e) We have $3/4 + 1/4 = 1$, which it must since the events in parts (a) and (d) are complementary events.

11.5.9 $E(X_3 X_5) = 61.75$

11.5.11 (a) $P(X_{10} > 250) = \Phi(-20/1\sqrt{10})$ (b) $P(X_{10} > 250) = \Phi(-20/4\sqrt{10})$ (c) $P(X_{10} > 250) = \Phi(-20/10\sqrt{10})$ (d) $P(X_{10} > 250) = \Phi(-20/100\sqrt{10})$

11.6.1 (a) $e^{-14}14^{13}/13!$ (b) $e^{-35}35^{3}/3!$ (c) $e^{-42}42^{20}/20!$ (d) $e^{-350}350^{340}/340!$ (e) 0 (f) $(e^{-14}14^{13}/13!)\,(e^{-21}21^{7}/7!)$ (g) 0

11.6.3 $P(N_2 = 6) = e^{-2/3}(2/3)^6/6!$, $P(N_3 = 5) = e^{-3/3}(3/3)^5/5!$

11.6.5 $P(N_{2.6} = 2 \mid N_{2.9} = 2) = (2.6/2.9)^2$